7 Polynomials

What You'll Learn

To use different tools to simplify algebraic expressions and to solve equations

And Why

To provide further strategies for problem solving

Key Words

- zero pair
- term
- like terms
- polynomial
- constant term
- distributive law
- monomial
- coefficient

Project Link

- It's All in the Game

Reading and Writing in Math

Doing Your Best on a Test

Taking a math test may be stressful.

Here are some tips that can help you do your best without getting too stressed.

Getting Ready

– Find out what will be covered on the test.
– Give yourself lots of time to review.
– Look over your notes and examples. Make sure they are complete.
– Get help with difficult questions.
– Make study notes. Use a mind map or a Frayer model.

When You First Get the Test

– Listen to the instructions carefully.
– Ask questions about anything that is not clear.
– Look over the whole test before you start.
– Write any formulas you have memorized at the top of the page before you begin.

Answering Multiple-Choice Questions

– Read the question and the possible answers carefully.
– Eliminate any possible answer that cannot be correct.
– Answer the question, even if you have to guess.

Answering Other Questions

– Make three passes through the test:
  1. Answer all the quick and easy questions.
  2. Answer all the questions that you know how to do but take some work.
  3. Work through the more difficult questions.
– Read the questions carefully.
  Underline or highlight important information.
  Look for key words such as:
  compare, describe, determine, explain, justify
– Pace yourself! Don't get stuck in one place.

When You Have Finished

– Make sure you have answered all the questions.
– Look over your written solutions. Are they reasonable?
  Did you show all your thinking and give a complete solution?

7.1 Like Terms and Unlike Terms

Algebra tiles are integer tiles and variable tiles.

Positive tiles

represents 1

represents x

represents x^2

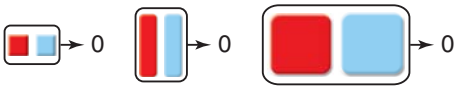
Negative tiles

represents -1

represents $-x$

represents $-x^2$

Any two opposite tiles add to 0. They form a **zero pair**.



Investigate Using Algebra Tiles to Model Expressions

Work with a partner. Repeat each activity 5 times.

Use at least 2 different kinds of red tiles. Vary the number and kind of tiles used.

- Place some red algebra tiles on your desk.
  Group like tiles together.
  Describe the collection of tiles in words and as an algebraic expression.
- Place some red and blue tiles on your desk.
  Group like tiles together.
  Remove any zero pairs.
  Describe the remaining collection of tiles in words and as an algebraic expression.

Reflect

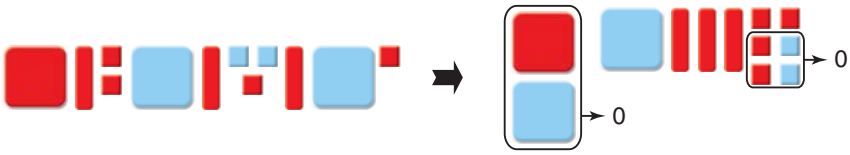
- What are like tiles?
  Why can they be grouped together?
- How would you represent like tiles algebraically?
- Why can you remove zero pairs?
- How do you know you have given the simplest name to a collection of tiles?

Connect the Ideas

To organize a collection of algebra tiles, we group like tiles.

There are two x^2-tiles, three x-tiles, and five 1-tiles.
These tiles represent the expression $2x^2 + 3x + 5$.

When a collection contains red and blue tiles,
we group like tiles and remove zero pairs.



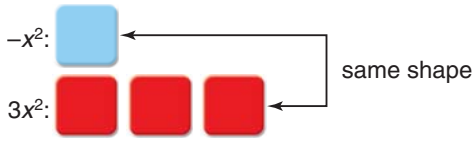
One $-x^2$-tile, three x-tiles, and two 1-tiles are left.
We write $-1x^2 + 3x + 2$.
We could also write $-x^2 + 3x + 2$.

The expression $-x^2 + 3x + 2$ has 3 **terms**: $-x^2$, $3x$, and 2

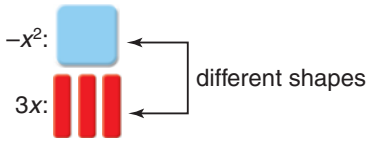
Terms are numbers, variables, or the products of numbers and variables.
Terms that are represented by like tiles are called **like terms**.

The numerical part of a term is its **coefficient.** The coefficient of $3x^2$ is 3. It tells us that there are three x^2-tiles.

$-x^2$ and $3x^2$ are like terms.
Each term is modelled with x^2-tiles.
In each term, the variable x is raised to the exponent 2.

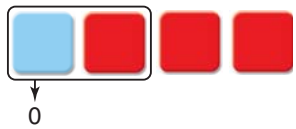


$-x^2$ and $3x$ are unlike terms.
Each term is modelled with different sized tiles.
Each term has the variable x, but the exponents are different.



An expression is simplified when all like terms are combined, and any zero pairs are removed.

$-x^2 + 3x^2$ simplifies to $2x^2$.



$-x^2 + 3x$ cannot be simplified.

Practice

1. Which expression does each group of algebra tiles represent?

a) 

b) 

c) 

d) 

e) 

f) 

2. Use algebra tiles to model each expression. Sketch the tiles you used.

a) $x - 5$ b) $2x^2 + 3$ c) $-x + 3$

d) $x^2 - 4x$ e) $4x^2 - 3x + 2$ f) $-2x^2 - x - 5$

3. The diagram shows the length and width of the 1-tile, x-tile, and x^2-tile.

Determine the area of each tile.
Use your answer to explain the name of the tile.

4. Use algebra tiles to show $2x$ and $-4x$.
Sketch the tiles you used.
Are $2x$ and $-4x$ like terms? Explain.

5. Use algebra tiles to show $3x$ and $3x^2$.
Sketch the tiles you used.
Are $3x$ and $3x^2$ like terms? Explain.

6. a) Identify terms that are like $3x$:
$-5x, 3x^2, 3, 4x, -11, 9x^2, -3x, 7x, x^3$

b) Identify terms that are like $-2x^2$:
$2x, -3x^2, 4, -2x, x^2, -2, 5, 3x^2$

c) Explain how you identified like terms in parts a and b.

7. In each part, combine like terms. Write the simplified expression.

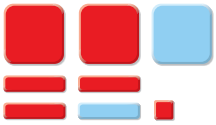
a) 

b) 

c) 

d) 

e) 

f) 

8. Combine like terms. Use algebra tiles.

a) $3x + 1 + 2x + 3$ b) $3x^2 - 2x + 5x + 4x^2$ c) $2x^2 + 3x - 2x + 4 - x^2$

9. Write an expression with 5 terms that has only 2 terms when it is simplified.

When we need many tiles to simplify an expression, it is easier to use paper and pencil.

Example

Simplify.

$15x^2 - 2x + 5 + 10x - 8 - 9x^2$

Visualize algebra tiles.

Solution

$$\begin{aligned} & 15x^2 - 2x + 5 + 10x - 8 - 9x^2 \\ &= 15x^2 - 9x^2 - 2x + 10x + 5 - 8 \\ &= 6x^2 + 8x - 3 \end{aligned}$$

Group like terms.
To combine like terms, add their coefficients.

10. a) Simplify each expression.

i) $-2 + 4x - 2x + 3$

ii) $2x^2 - 3x + 4x^2 - 6x$

iii) $3x^2 + 4x + 2 + x^2 + 2x + 1$

iv) $x^2 - 4x + 3 - 2 + 5x - 4x^2$

b) Create an expression that cannot be simplified. Explain why it cannot be simplified.

Which tools could you use to help you?

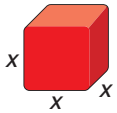
11. **Assessment Focus**

a) Determine the volume of this cube.

b) Use the volume to suggest a name for the cube.

c) Simplify. How can you use cubes to do this?

i) $x^3 + 2x^3 + 5x^3$ ii) $3x^3 + 3x + x^3 + 5x$ iii) $5 - 2x^3 + 3x^2 + 5x^3$

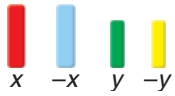


12. **Take It Further** Many kits of algebra tiles contain a second variable tile called a *y*-tile.

a) Why does a *y*-tile have a different length than an *x*-tile?

b) Sketch algebra tiles to represent $2x - 5y - 1 + 4y - 7x + 4$.

c) Write the expression in part b in simplest form. How do you know it is in simplest form?



In Your Own Words

Create a Frayer model for like terms.
Explain how like terms can be used to write an expression in simplest form.
Use diagrams and examples in your explanation.

7.2 Modelling the Sum of Two Polynomials

Expressions such as $3x^2$, $2x + 1$, and $4x^2 - 2x + 3$ are polynomials. A **polynomial** can be one term, or the sum or difference of two or more terms.

Investigate Using Algebra Tiles to Add Polynomials

Work with a partner.
You will need algebra tiles and a paper bag.

- Put the red algebra tiles in a bag.
  Take turns to remove a handful of tiles,
  then write the polynomial they model.
  Add the two polynomials.
  Write the resulting polynomial.
- Repeat the previous activity with red and blue tiles.

Reflect

- How do you model addition with algebra tiles?
- Which terms can be combined when you add polynomials?
  Why can these terms be combined?
- Compare the original polynomials and the polynomial for their sum.
  What remains the same? What changes? Explain.

Connect the Ideas

To add the polynomials $2x^2 - 5x + 3$ and $x^2 + 3x - 2$, we write $(2x^2 - 5x + 3) + (x^2 + 3x - 2)$.

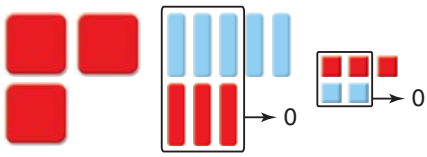
Use algebra tiles

Model each polynomial.

$2x^2 - 5x + 3$

$x^2 + 3x - 2$

Combine like terms.



The remaining tiles represent $3x^2 - 2x + 1$.
So, $(2x^2 - 5x + 3) + (x^2 + 3x - 2) = 3x^2 - 2x + 1$

Use paper and pencil

Group like terms and simplify.

$(2x^2 - 5x + 3) + (x^2 + 3x - 2)$	Remove the brackets.
$= 2x^2 - 5x + 3 + x^2 + 3x - 2$	Group like terms.
$= 2x^2 + x^2 - 5x + 3x + 3 - 2$	Combine like terms.
$= 3x^2 - 2x + 1$	

You can also add polynomials vertically.
$(2x^2 - 5x + 3) + (x^2 + 3x - 2)$

Align like terms.

Combine like terms.

$$\begin{array}{r} 2x^2 - 5x + 3 \\ +\quad x^2 + 3x - 2 \\ \hline 3x^2 - 2x + 1 \end{array}$$

Practice

1. Which polynomial sum does each set of tiles represent?

a) 

b) 

2. Use algebra tiles to add these polynomials.
Sketch the tiles you used.

a) $(2 + x^2) + (-3x^2 - 5)$
b) $(-2 - x^2) + (3x^2 + 5)$
c) $(-2 + x^2) + (-3x^2 + 5)$
d) $(2 - x^2) + (-3x^2 + 5)$

3. Add. Use algebra tiles if it helps.

a) $(2x - 3) + (4x + 5)$
b) $(9x - 5) + (7 - 6x)$
c) $(-x + 2) + (5x - 1)$
d) $(3x + 3) + (-4x - 5)$
e) $(8x - 2) + (-6x - 6)$
f) $(3x - 2) + (-x + 6)$

4. Add. Use algebra tiles if it helps.

a) $(3x^2 - 4x) + (5x^2 + 7x - 1)$
b) $(-3x^2 - 4x + 2) + (5x^2 - 8x - 7)$
c) $(x^2 - 2x - 1) + (3x^2 + x + 2)$
d) $(x^2 - 5) + (-2x^2 + 2x - 3)$
e) $(3x^2 - x + 5) + (-8x^2 + 3x - 1)$
f) $(3x^2 + 2x + 1) + (-2x^2 + 3x - 4)$

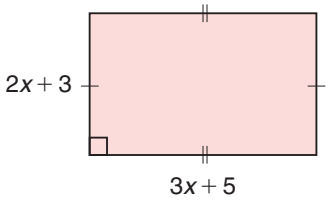
5. Add: $(3x^3 + 2x^2 - x + 1) + (-2x^3 - 6x^2 + 4x - 3)$
Explain how you did it.

Which tools could you use to help you?

Polynomials can be used to represent side lengths of figures.

Example

Write a simplified expression for the perimeter of this rectangle.



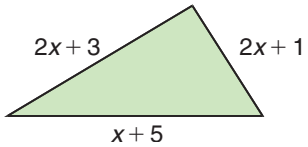
Solution The perimeter is the sum of the measures of the four sides.

$$\begin{array}{r} 2x + 3 \\ + \; 2x + 3 \\ + \; 3x + 5 \\ + \; 3x + 5 \\ \hline 10x + 16 \end{array}$$

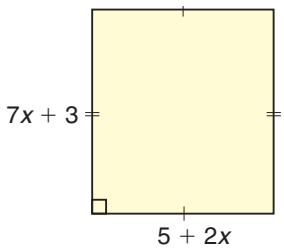
The perimeter is $10x + 16$.

6. Write an expression for the perimeter of each figure.
Simplify the expression.

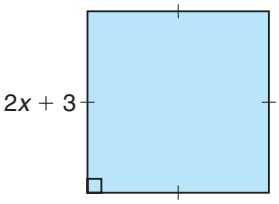
a)



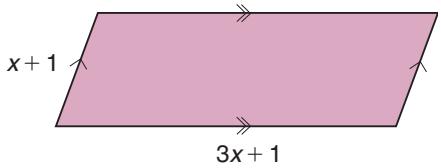
b)



c)



d)



Need Help?
Read the Example on page 259.

7. Choose one of the figures in question 6.
a) Evaluate the unsimplified expression for the perimeter when $x = 8$.
b) Evaluate the simplified expression for the perimeter when $x = 8$.
c) Is it better to substitute into an expression before it is simplified or after it is simplified? Explain.

8. Create a polynomial that is added to $2x^2 + 3x + 7$ to get each sum.
a) $4x^2 + 5x + 9$ **b)** $3x^2 + 4x + 8$
c) $2x^2 + 3x + 7$ **d)** $x^2 + 2x + 6$
e) $x + 5$ **f)** 4

9. **Assessment Focus** Two polynomials are added.
Their sum is $3x^2 - 2x + 5$.
Write two polynomials that have this sum.
How many different pairs of polynomials can you find?
Which tools did you use to help you?

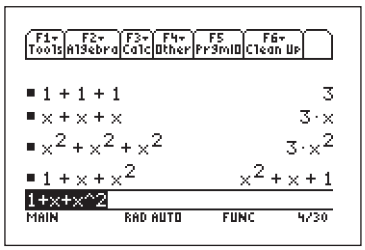
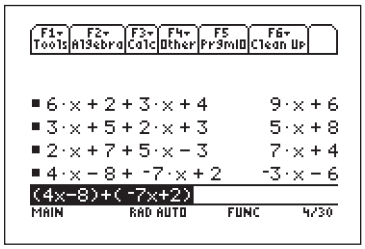
10. **Take It Further** The sum of 2 polynomials is $2x^2 - 7x + 3$.
One polynomial is $3x^2 - 5x - 2$.
What is the other polynomial?
Explain how you found it.

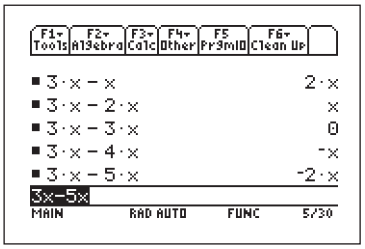
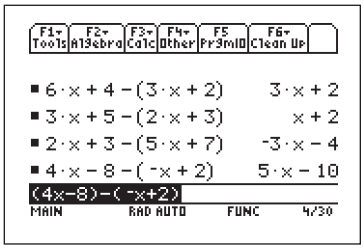
In Your Own Words

Write two things you know about adding polynomials.
Use an example to illustrate.

Using CAS to Add and Subtract Polynomials

You will need a TI-89 calculator.

Step 1 ➢ Clear the home screen: [HOME] [F1] 8 [CLEAR] ➢ Enter $1 + 1 + 1$: 1 [+] 1 [+] 1 [ENTER] ➢ Enter $x + x + x$: [X] [+] [X] [+] [X] [ENTER] ➢ Enter $x^2 + x^2 + x^2$: [X][^]2 [+] [X][^]2 [+] [X][^]2 [ENTER] ➢ Enter $1 + x + x^2$: 1 [+] [X] [+] [X][^]2 [ENTER]	At the end of Step 1, your screen should look like this:  	Record the first 3 lines of the screen on paper. What patterns do you see? Use algebra tiles to explain the patterns. Record the last line of the screen on paper. Use algebra tiles to explain why the terms could not be simplified.
Step 2 To add $6x + 2$ and $3x + 4$, we write: $(6x + 2) + (3x + 4)$ ➢ Clear the home screen: [HOME] [F1] 8 [CLEAR] ➢ Enter $(6x + 2) + (3x + 4)$: [(]6[X] [+] 2[)] [+] [(]3[X] [+] 4[)] [ENTER] ➢ Enter $(3x + 5) + (2x + 3)$: [(]3[X] [+] 5[)] [+] [(]2[X] [+] 3[)] [ENTER] ➢ Enter $(2x + 7) + (5x - 3)$. ➢ Enter $(4x - 8) + (-7x + 2)$.	At the end of Step 2, your screen should look like this:  	Record the screen on paper. Which terms are combined when polynomials are added? What remains the same when the terms are combined? What changes when the terms are combined?
Step 3 Use paper and pencil to add these polynomials: ➢ $(5x + 6) + (8x + 3)$ ➢ $(3x - 3) + (2x + 5)$ Use CAS to check your work.		Explain any mistakes you made. How could you avoid making the same mistakes in the future?

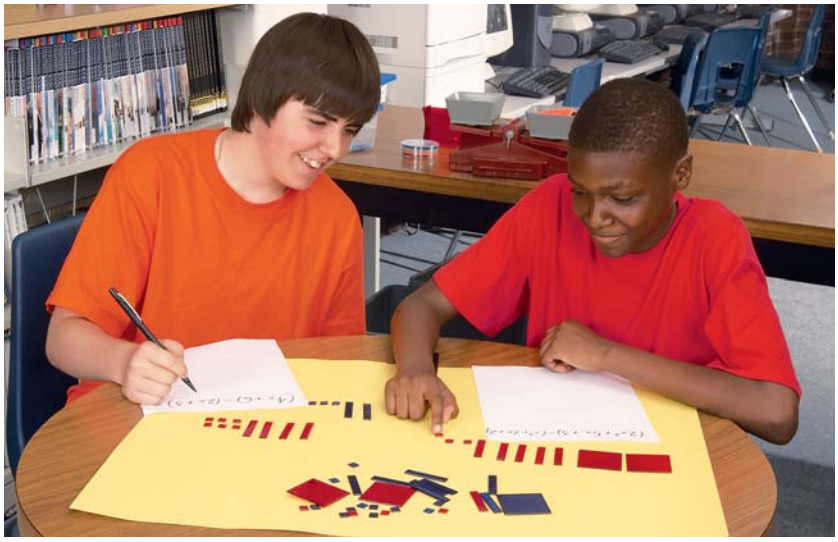
Step 4 ➢ Clear the home screen: [HOME] [F1] 8 [CLEAR] ➢ Enter $3x - x$: 3[X] [−] [X] [ENTER] ➢ Enter $3x - 2x$: 3[X] [−] 2[X] [ENTER] ➢ Enter $3x - 3x$: 3[X] [−] 3[X] [ENTER] ➢ Continue the pattern until your screen is like the one at the right.	At the end of Step 4, your screen should look like this:  	Record the screen on paper. What remains the same when terms are subtracted? What changes when terms are subtracted?
Step 5 To subtract $3x + 2$ from $6x + 4$, we write: $(6x + 4) - (3x + 2)$ ➢ Clear the home screen: [HOME] [F1] 8 [CLEAR] ➢ Enter $(6x + 4) - (3x + 2)$: [(]6[X] [+] 4[)] [−] [(]3[X] [+] 2[)] [ENTER] ➢ Enter $(3x + 5) - (2x + 3)$: [(]3[X] [+] 5[)] [−] [(]2[X] [+] 3[)] [ENTER] ➢ Enter $(2x + 3) - (5x + 7)$. ➢ Enter $(4x - 8) - (-x + 2)$.	At the end of Step 5, your screen should look like this:  	Record the screen on paper. Explain each result.
Step 6 Use CAS to subtract these polynomials: ➢ $(x - 8) - (2x + 3)$ ➢ $(-3x + 5) - (-2x - 1)$		Explain any mistakes you made. How could you avoid making the same mistakes in the future?

7.3 Modelling the Difference of Two Polynomials

Investigate Using Algebra Tiles to Subtract Polynomials

Work with a partner.
You will need algebra tiles.

➢ Subtract.
$6 - 4$
$5x - 2x$
$(4x + 6) - (2x + 3)$
$(2x^2 + 5x + 3) - (x^2 + 2x + 2)$



➢ Subtract.
Add zero pairs so you have enough tiles to subtract.
$4 - 8$
$2x - 3x$
$(2x + 4) - (3x + 8)$
$(x^2 + 2x + 1) - (3x^2 + 4x + 3)$

Reflect

➢ How did you use algebra tiles to subtract polynomials?

➢ Why can you use zero pairs to subtract?

➢ How do you know how many zero pairs you need, to be able to subtract?

Connect the Ideas

To determine: $(-x^2 - 2x + 1) - (2x^2 + 4x - 3)$

Use algebra tiles to model $-x^2 - 2x + 1$:

To subtract $2x^2 + 4x - 3$, we need to take away 2 red x^2-tiles, 4 red x-tiles, and 3 blue 1-tiles. But we do not have these tiles to take away. So, we add zero pairs that include these tiles.

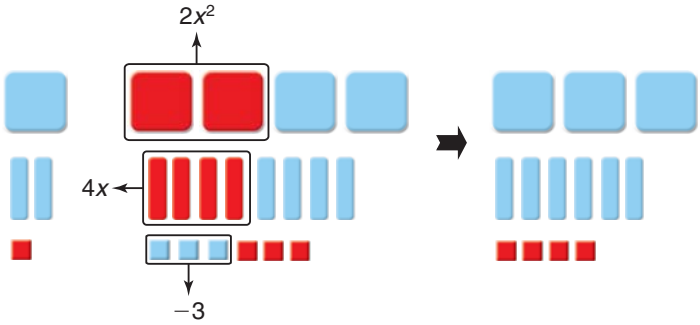
Adding these tiles has not changed the polynomial represented.

Zero pair for $2x^2$:

Zero pair for $4x$:

Zero pair for -3:

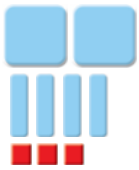
Now we can take away the tiles for $2x^2 + 4x - 3$.



The remaining tiles represent $-3x^2 - 6x + 4$.
So, $(-x^2 - 2x + 1) - (2x^2 + 4x - 3) = -3x^2 - 6x + 4$

When we subtracted $2x^2 + 4x - 3$, we ended up adding a polynomial with opposite signs. That is, we added the opposite polynomial, $-2x^2 - 4x + 3$:

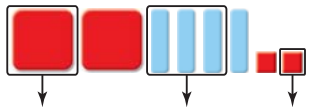
We get the opposite polynomial by changing all the signs of the coefficients of a polynomial.



This is true in general.
To subtract a polynomial, we add its opposite.

Practice

1. Which polynomial difference is represented by these tiles?



2. Simplify these polynomials. Use algebra tiles.

a) $(5x + 3) - (3x + 2)$ **b)** $(5x + 3) - (3x - 2)$

c) $(5x + 3) - (-3x + 2)$ **d)** $(5x + 3) - (-3x - 2)$

3. Simplify these polynomials. Use algebra tiles.

a) $(3x^2 + 2x + 4) - (2x^2 + x + 1)$ **b)** $(3x^2 - 2x + 4) - (2x^2 - x + 1)$

c) $(3x^2 - 2x - 4) - (-2x^2 + x - 1)$ **d)** $(-3x^2 + 2x - 4) - (2x^2 - x - 1)$

4. Show the opposite of each polynomial using algebra tiles.
Write the opposite polynomial.

a) $3x - 2$ **b)** $-2x^2 + 3x$ **c)** $4x^2 - 7x + 6$ **d)** $-x^2 + 5x - 4$

We can subtract polynomials using paper and pencil.

Example

Subtract: $(5x^2 - 2x + 4) - (7x^2 + 10x - 8)$

Solution

$(5x^2 - 2x + 4) - (7x^2 + 10x - 8)$

To subtract $7x^2 + 10x - 8$, add its opposite $-7x^2 - 10x + 8$.

$$\begin{aligned}
&(5x^2 - 2x + 4) - (7x^2 + 10x - 8)\\
&= 5x^2 - 2x + 4 + (-7x^2 - 10x + 8)\\
&= 5x^2 - 2x + 4 - 7x^2 - 10x + 8\\
&= 5x^2 - 7x^2 - 2x - 10x + 4 + 8\\
&= -2x^2 - 12x + 12
\end{aligned}$$

To check, add the difference to the second polynomial:

$$\begin{aligned}
&(-2x^2 - 12x + 12) + (7x^2 + 10x - 8)\\
&= -2x^2 + 7x^2 - 12x + 10x + 12 - 8\\
&= 5x^2 - 2x + 4
\end{aligned}$$

The sum is equal to the first polynomial.
So, the difference is correct.

5. Simplify.

a) $(x + 7) - (x + 5)$ **b)** $(x + 7) - (x - 5)$

c) $(x + 7) - (-x + 5)$ **d)** $(x + 7) - (-x - 5)$

6. Simplify. Check your answers by adding.

a) $(2x^2 - 3) - (x^2 + 1)$ **b)** $(3x^2 + 2x) - (2x^2 + x)$

c) $(7 - 4x^2) - (8 - 2x^2)$ **d)** $(5x - 7x^2) - (2x^2 + 2x)$

Which tools can you use to help you?

7. Simplify. How could you check your answers?

a) $(3x^2 + x - 1) - (x^2 - 2x + 5)$

b) $(x^2 - x + 1) - (x^2 + x - 1)$

c) $(2x^2 + x - 3) - (x^2 - 3x + 4)$

d) $(x - x^3 + 5) - (7 - x + x^3)$

e) $(7 + 3x - 2x^3) - (4 - 3x + 3x^3)$

f) $(2x^2 - 3x - 5) - (2x^2 - 3x - 5)$

8. **Assessment Focus** John subtracted these polynomials:

$(2x^2 - 4x + 6) - (3x^2 + 2x - 4)$

a) Explain why his solution is incorrect.

$$\begin{aligned} &(2x^2 - 4x + 6) - (3x^2 + 2x - 4) \\ &= 2x^2 - 4x + 6 - 3x^2 + 2x - 4 \\ &= 2x^2 - 3x^2 - 4x + 2x + 6 - 4 \\ &= -x^2 - 2x + 2 \end{aligned}$$

b) What is the correct answer? Show your work.

c) How could you check your answer?

9. **a)** Simplify.

i) $(3x^2 + x) - (2x^2 - 3x)$

ii) $(2x^2 - 3x) - (3x^2 + x)$

iii) $(4x^3 - 5) - (7 + 2x^3)$

iv) $(7 + 2x^3) - (4x^3 - 5)$

v) $(2x - x^3) - (-x^3 + 2x)$

vi) $(-x^3 + 2x) - (2x - x^3)$

b) What patterns do you see in the answers in part a? Explain.

c) Write two polynomials.
Subtract them in different orders.
What do you notice?

10. **a)** **i)** Write a polynomial.
ii) Write the opposite polynomial.

b) Subtract the two polynomials in part a.
What do you notice about your answer?

c) Compare your answer with those of your classmates.
Is there a pattern? Explain.

11. **Take It Further** One polynomial is subtracted from another.
The difference is $-2x^2 + 4x - 5$.
Write two polynomials that have this difference.
How many different pairs of polynomials can you find?

In Your Own Words

What did you find difficult about subtracting two polynomials?
Use examples to show how you overcame this difficulty.

I Have... Who Has?

GAME

A game card looks like this:

> *I have ...*
> *$x + 1$*
> *Who has ...*
> *$5x - 2 + 3x - 2$ simplified?*

Materials
2 sets of game cards: green for Team A and yellow for Team B

- Divide the class into two teams.
  Toss a coin to see which team starts.
  Distribute a game card to each student.
- A student from the team that won the coin toss starts.
  The student reads her card.
  "I have ... $x + 1$.
  Who has ... $5x - 2 + 3x - 2$ simplified?"
- All students on the second team simplify the expression.
  The student whose card has the solution $8x - 4$ says:
  "I have ... $8x - 4$.
  Who has ... the opposite of $3x^2 - 4x + 1$?"
- A team makes a mistake when a student does not respond or responds incorrectly.
- The game ends when all the cards have been called.
  The team with the fewer mistakes wins.

Mid-Chapter Review

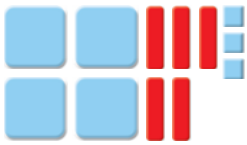
7.1 **1.** Renata simplified an expression and got $-7 + 4x^2 + 2x$.
Her friend simplified the same expression and got $4x^2 + 2x - 7$.
Renata thinks her answer is wrong.
Do you agree? Explain.

2. Which expression does each group of tiles represent?

a) 

b) 

c) 

d) 

3. Combine like terms.

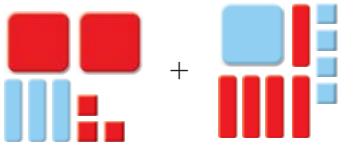
a) $5x + 6 + 3x$
b) $3x - x + 2$
c) $2x^2 - 6 + x^2 + 3$
d) $-5 + x + 3 - 2x$
e) $x - 4 + 6 - 5x$
f) $3x^3 - x + 2x^3 + 3x$
g) $4x^2 + 3x + 5 - 5x^2 - 2x - 7$

Which tools could you use to help you?

4. Cooper thinks $5x + 2$ simplifies to $7x$.
Is he correct? Explain.
Sketch algebra tiles to support your explanation.

5. Write an expression with 4 terms that has only 1 term when it is simplified.
Explain your thinking.

7.2 **6.** Which polynomial sum do these tiles represent?



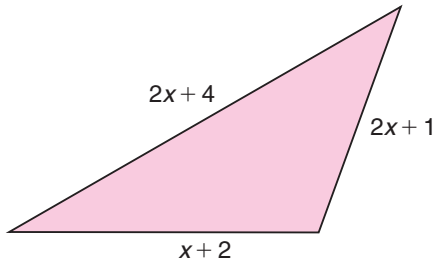
7. Add.

a) $(5x + 4) + (3x + 5)$
b) $(4x - 3) + (5 - 2x)$
c) $(7x^2 - 4x) + (3x^2 - 2)$
d) $(5x^2 + 2x + 1) + (x^2 - 3x - 5)$
e) $(-2x^2 - x + 1) + (x^2 + 4x - 2)$
f) $(3x^2 + 4x + 2) + (-4x^2 - 3x - 5)$

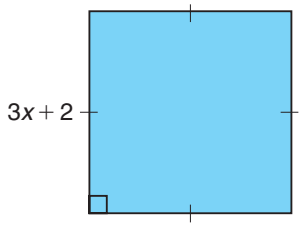
8. Which polynomial must be added to $5x^2 + 3x - 2$ to get $7x^2 + 5x + 1$?

9. Write a simplified expression for the perimeter of each figure.

a)



b)



7.3 **10.** Write the opposite polynomial.

a) $x + 3$
b) $3x^2 - 5x$
c) $2x^2 + 3x + 7$
d) $-5x^2 - 2x - 1$

11. Subtract.
How could you check your answers?

a) $(x + 3) - (x - 4)$
b) $(7x^2 - 4x) - (x^2 - 2x)$
c) $(3x^2 - 2x + 7) - (5x^2 - 3x - 2)$
d) $(-2x^2 - 4x + 1) - (3x^2 + 2x - 1)$
e) $(4x^2 - 2x - 1) - (4x^2 - 5x - 8)$

7.4 Modelling the Product of a Polynomial and a Constant Term

Algebra tiles are area tiles.
The name of the tile tells its area.

Tile	Dimensions (units)	Area (square units)
1-tile	1, 1	$(1)(1) = 1$
x-tile	1, x	$(1)(x) = x$
x^2-tile	x, x	$(x)(x) = x^2$

Investigate Using Algebra Tiles to Multiply

Work with a partner.
Use algebra tiles.
In each case, sketch the tiles you used.

Remember that multiplication is repeated addition.

- Display the polynomial $3x + 1$.
  Use more tiles to display the product $2(3x + 1)$.
  Combine like tiles.
  Write the resulting product as a polynomial.
- Display the product $3(3x + 1)$.
  Combine like tiles.
  Write the resulting product as a polynomial.
- Follow the steps above for these products:
  $4(3x + 1)$
  $5(3x + 1)$

Reflect

- What patterns do you see in the polynomials and their products?
- Use these patterns to write the product $6(3x + 1)$ as a polynomial.
- Use the patterns to find a way to multiply without using algebra tiles.

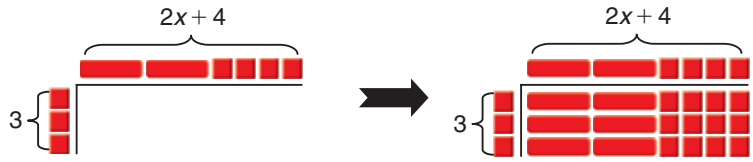
Connect the Ideas

Use a model

We can model $3(2x + 4)$ as the area of a rectangle in two ways. The rectangle has length $2x + 4$ and width 3.

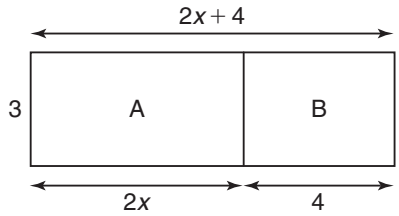
- Use algebra tiles to label the length and width.

Use the labels as a guide. Fill in the rectangle.



Six x-tiles and twelve 1-tiles have a combined area of $6x + 12$.
So, $3(2x + 4) = 6x + 12$

- Draw and label a rectangle.
  The length of the rectangle is $2x + 4$.
  The width of the rectangle is 3.
  So, the area of the rectangle is $3(2x + 4)$.



We divide the rectangle into two rectangles.
Rectangle A has area: $3(2x) = 6x$
Rectangle B has area: $3(4) = 12$
The total area is: $6x + 12$
So, $3(2x + 4) = 6x + 12$

Use paper and pencil

We can also determine $3(2x + 4)$ using paper and pencil.

Each term in the brackets is multiplied by the constant term 3 outside the brackets.
This process is called *expanding*.

$$\begin{aligned} 3(2x + 4) &= 3(2x) + 3(4) \\ &= 6x + 12 \end{aligned}$$

In algebra, a term without a variable is called a **constant term**.

This illustrates the **distributive law**.
When we use the distributive law to multiply a polynomial by a constant term, we *expand* the product.

Practice

1. Write the product modelled by each set of tiles. Determine the product.

a) 

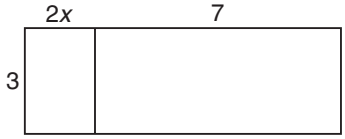
b) 

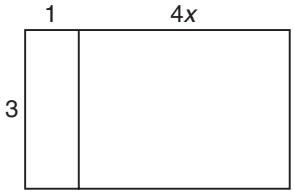
c) 

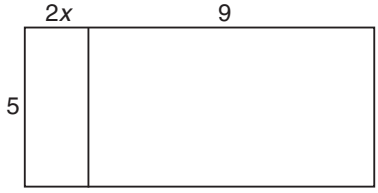
2. Use algebra tiles to determine each product. Sketch the tiles you used.

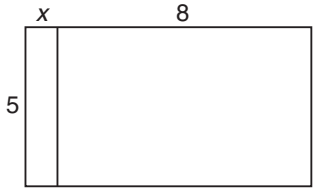
a) $2(4x + 1)$ b) $2(3x + 1)$ c) $5(2x + 3)$ d) $4(4x + 3)$

3. Write the product modelled by each figure. Determine the product.

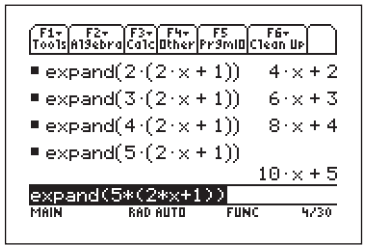
a) 

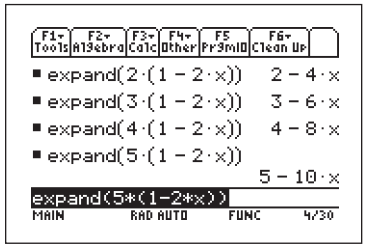
b) 

c) 

d) 

4. Use the calculator screens below.
What patterns do you see? Explain each pattern.
Write the next 2 lines in each pattern.

a) 

b) 

We can also use paper and pencil to expand a product.

Example

Expand: $-3(-2x^2 + 3x - 4)$

Solution

$-3(-2x^2 + 3x - 4)$

Multiply each term in the brackets by -3.

$$\begin{aligned} -3(-2x^2 + 3x - 4) &= (-3)(-2x^2) + (-3)(3x) + (-3)(-4) \\ &= (+6x^2) + (-9x) + (+12) \\ &= 6x^2 - 9x + 12 \end{aligned}$$

5. Expand: $-100(4x^2 + x - 4)$
Visualize algebra tiles. Are they useful to determine this product?
Explain.

6. Jessica expands $3(x^2 + 4x - 2)$ and gets $3x^2 + 4x - 2$.
Choose a tool.
Use the tool to explain why Jessica's answer is incorrect.

7. Multiply.

a) $7(3x - 1)$ **b)** $3(4x - 5)$ **c)** $2(6x - 4)$
d) $5(5x^2 - 3x)$ **e)** $4(-2x^2 + 3)$ **f)** $9(x^2 + x - 6)$

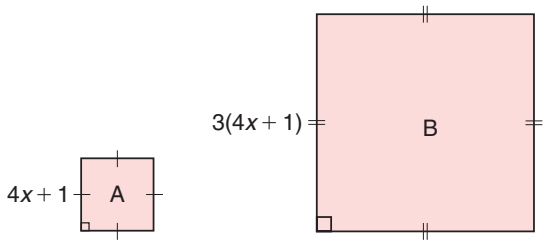
8. Expand.

a) $-2(4x + 2)$ **b)** $3(-5x^2 + 3)$ **c)** $2(-x - 4)$
d) $-7(x^2 - 5)$ **e)** $-2(-5x^2 + x)$ **f)** $6(3 - 2x)$

9. **Assessment Focus** Multiply. Which tools did you use? Explain.

a) $-2(x^2 - 2x + 4)$ **b)** $-3(-x^2 + 3x - 7)$ **c)** $2(4x^3 - 2x^2 - x)$
d) $8(3x^3 + 2x^2 - 3)$ **e)** $-6(2x^2 - x + 5)$ **f)** $4(x^2 - 3x - 3)$

10. Square A has side length $4x + 1$.
Square B has side length that is 3 times as great as that of Square A.



a) Write an expression for the perimeter of each square.
Simplify each expression.
b) What is the difference in perimeters?

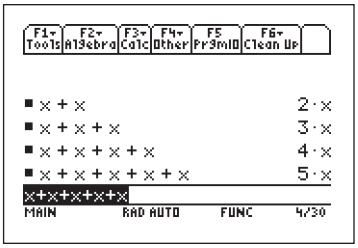
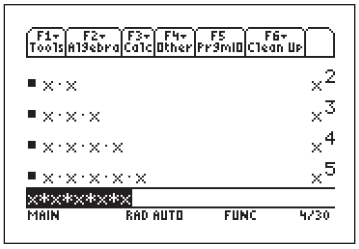
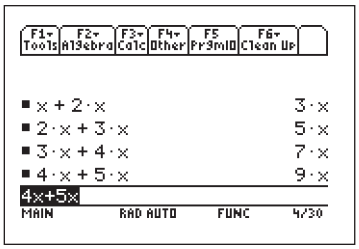
11. **Take It Further** Explain how you could use algebra tiles to multiply a polynomial by a negative constant term.
Illustrate with an example.

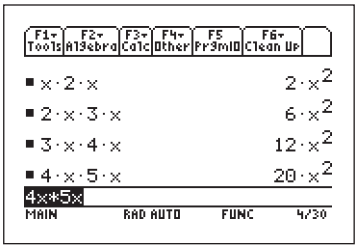
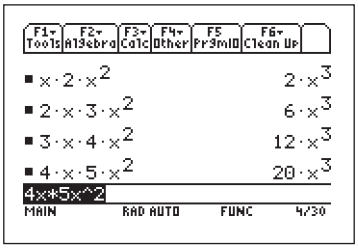
In Your Own Words

When can you use algebra tiles to determine the product of a constant term and a polynomial?
When do you use paper and pencil?
Include examples in your explanation.

Using CAS to Investigate Multiplying Two Monomials

You will need a TI-89 calculator.

Step 1 ➢ Clear the home screen: [HOME] [F1] 8 [CLEAR] ➢ Enter $x + x$: [X] [+] [X] [ENTER] ➢ Enter $x + x + x$: [▶] [+] [X] [ENTER] ➢ Continue the pattern of adding x until your screen is like the one at the right.	At the end of Step 1, your screen should look like this:  	Record the screen on paper. Write the next 2 lines in the pattern.
Step 2 ➢ Clear the home screen: [HOME] [F1] 8 [CLEAR] ➢ Enter x times x: [X] [×] [X] [ENTER] ➢ Enter x times x times x: [▶] [×] [X] [ENTER] ➢ Continue the pattern of multiplying by x until your screen is like the one at the right.	At the end of Step 2, your screen should look like this:  	Record the screen on paper. Write the next 2 lines in the pattern. Compare the results of Step 1 and Step 2. How are they different? Explain.
Step 3 ➢ Clear the home screen: [HOME] [F1] 8 [CLEAR] ➢ Enter $x + 2x$: [X] [+] 2[X] [ENTER] ➢ Enter $2x + 3x$: 2[X] [+] 3[X] [ENTER] ➢ Continue the pattern until your screen is like the one at the right.	At the end of Step 3, your screen should look like this:  	Record the screen on paper. Write the next 2 lines in the pattern.

Step 4 ➢ Clear the home screen: [HOME] [F1] 8 [CLEAR] ➢ Multiply x by $2x$: [X] [×] 2[X] [ENTER] ➢ Multiply $2x$ by $3x$: 2[X] [×] 3[X] [ENTER] ➢ Continue the pattern until your screen is like the one at the right.	At the end of Step 4, your screen should look like this:  	Record the screen on paper. Write the next 2 lines in the pattern. A polynomial, such as $2x$ or $3x$, is called a monomial. How is multiplying monomials different from adding monomials?
Step 5 ➢ Clear the home screen: [HOME] [F1] 8 [CLEAR] ➢ Multiply x by $2x^2$: [X] [×] 2[X][^]2 [ENTER] ➢ Multiply $2x$ by $3x^2$: 2[X] [×] 3[X][^]2 [ENTER] ➢ Continue the pattern until your screen is like the one at the right.	At the end of Step 5, your screen should look like this:  	Record the screen on paper. Write the next 2 lines in the pattern. Explain the pattern.
Step 6 Use CAS to multiply. ➢ $(2x)(6x)$ ➢ $(2x)(-6x)$ ➢ $(-2x)(6x)$ ➢ $(-2x)(-6x)$ ➢ $(5x)(5x)$ ➢ $(-8x)(3x^2)$ ➢ $(-2x^2)(-3x)$		$(2x)(6x)$ means $2x$ multiplied by $6x$. How could you use paper and pencil to multiply two monomials?

7.5 Modelling the Product of Two Monomials

Investigate Using Algebra Tiles to Multiply Monomials

Use algebra tiles.
In each case, sketch the tiles you used.

Recall that $(x)(2x)$ means x times $2x$.

- Build a rectangle x units wide and $2x$ units long.
  Which tiles did you use?
  What is the area of the rectangle?
  Simplify: $(x)(2x)$
- Show the product: $(2x)(2x)$
  Simplify: $(2x)(2x)$
- Show the product: $(3x)(2x)$
  Simplify: $(3x)(2x)$
- Show the product: $(4x)(2x)$
  Simplify: $(4x)(2x)$
- Show the product: $(5x)(2x)$
  Simplify: $(5x)(2x)$

Reflect

- What patterns do you see in the terms and their products?
- Use these patterns to simplify: $(6x)(2x)$
- Use the patterns to find a way to multiply without using algebra tiles.

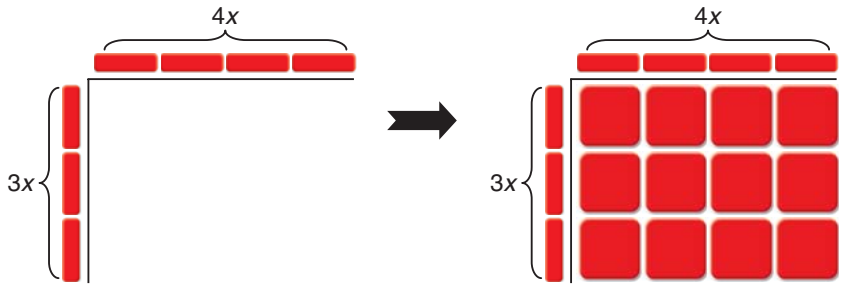
Connect the Ideas

A polynomial with one term is called a **monomial**.
Here are some monomials: $2x$, 5, -3, $6x^2$, $-x^3$

Use algebra tiles

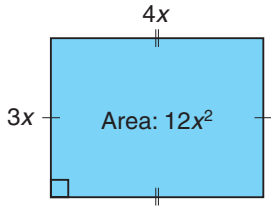
We can use algebra tiles to determine the product of some monomials.

To determine $(3x)(4x)$, draw two sides of a rectangle whose length is $4x$ and whose width is $3x$.



Use algebra tiles to fill the rectangle.
We need twelve x^2-tiles.
So, $(3x)(4x) = 12x^2$

Look at the product:
$(3x)(4x)$



The numbers 3 and 4 are coefficients.
To multiply two monomials,
multiply the coefficients: $(3)(4) = 12$
multiply the variables: $(x)(x) = x^2$
So, $(3x)(4x) = 12x^2$

Use paper and pencil

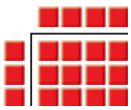
The term x has coefficient 1; that is, x can be written as $1x$.

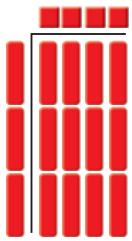
We cannot use algebra tiles to determine a product such as:
$(x)(2x^2)$
We use paper and pencil instead.
Multiply the coefficients: $(1)(2) = 2$
Multiply the variables: $(x)(x^2) = (x)(x)(x)$
$= x^3$

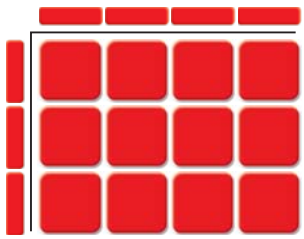
So, $(x)(2x^2) = 2x^3$

Practice

1. Write the product modelled by each set of tiles.
Determine the product.

a) 

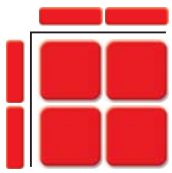
b) 

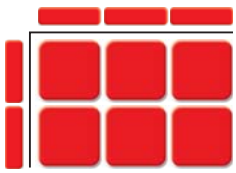
c) 

2. Look at the statements and products in question 1.
How are they alike?
How are they different?

3. Write the product modelled by each set of tiles.
Determine the product.

a) 

b) 

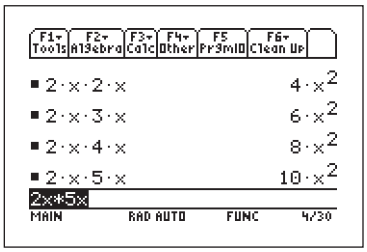
c) 

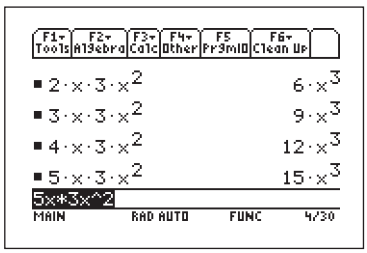
Having Trouble?
Read Connect the Ideas

4. Use algebra tiles to multiply.

a) $(x)(3x)$ b) $(x)(5x)$ c) $(3x)(5x)$ d) $(4x)(x)$
e) $(x)(4x)$ f) $(2x)(6x)$ g) $(7x)(2x)$ h) $(3x)(3x)$

5. Look at the calculator screens below.
What patterns do you see?
Explain each pattern.
Write the next 3 lines in each pattern.

a) 

b) 

6. Multiply.

a) $(2x^2)(3x)$ b) $(4x)(x^2)$ c) $(x)(3x^2)$ d) $(x^2)(x)$
e) $(6x^2)(4x)$ f) $(2x)(8x^2)$ g) $(3x^2)(8x)$ h) $(3x)(3x^2)$

We can use paper and pencil to multiply monomials when there is a negative coefficient.

Example

Multiply: $(-3x^2)(7x)$

Solution $(-3x^2)(7x)$

Multiply the coefficients: $(-3)(7) = -21$

Multiply the variables: $(x^2)(x) = (x)(x)(x)$
$= x^3$

So, $(-3x^2)(7x) = -21x^3$

7. Multiply.

a) $(-3)(2x)$ b) $(4x)(-4)$ c) $(8)(-x)$ d) $(5x)(-7)$
e) $(2x)(-9)$ f) $(5)(-2x)$ g) $(-9x)(9)$ h) $(-3x)(-6)$

8. Multiply.

a) $(2x^2)(-3x)$ b) $(-3x)(4x^2)$ c) $(-4x)(-5x^2)$ d) $(x^2)(-x)$
e) $(4x^2)(-8x)$ f) $(2x)(-2x^2)$ g) $(-x)(12x^2)$ h) $(-3x)(-12x^2)$

Which tools can you use to help you?

9. Students who have trouble with algebra often make these mistakes.

- They think $x + x$ equals x^2.
- They think $(x)(x)$ equals $2x$.

Choose a tool.

Use the tool to show how to get the correct answers.

10. **Assessment Focus**

a) Determine each product.

i) $(3x)(5x)$ ii) $(-3x)(5x)$ iii) $(3)(5x)$ iv) $(3x)(5)$
v) $(5x^2)(3)$ vi) $(5x^2)(-3)$ vii) $(-5x^2)(-3)$ viii) $(5x^2)(3x)$

b) List the products in part a that can be modelled with algebra tiles. Justify your list.

c) Sketch the tiles for one product you listed in part b.

11. **Take It Further** The product of two monomials is $36x^3$.

What might the two monomials be?

How many different pairs of monomials can you find?

In Your Own Words

Suppose you have to multiply two monomials.
How do you know when you can use algebra tiles?
When do you have to use paper and pencil? When can you use CAS?
Include examples in your answer.

7.6 Modelling the Product of a Polynomial and a Monomial

Investigate Using Models to Multiply a Polynomial by a Monomial

Work with a partner.

- Determine each product by using algebra tiles and by using an area model.
  $2(x + 3)$
  $2x(x + 3)$
- Write a different product that involves a monomial and a polynomial.
  Have your partner model your product with algebra tiles and with a rectangle.
- Repeat the previous step for 4 other products.
  Take turns to write the product, then model it.

Reflect

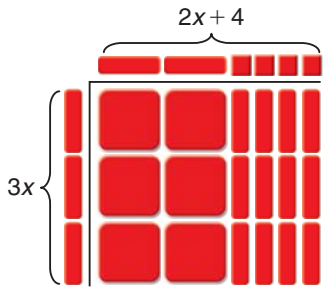
- What patterns do you see in the products you created and calculated?
- Did you write any products that could *not* be modelled with algebra tiles?
  If your answer is yes, how did you determine the product?
  If your answer is no, try to create a product that you cannot model with algebra tiles.
  Try to find a way to determine the product.

Connect the Ideas

There are 3 ways to expand $3x(2x + 4)$.

Use algebra tiles

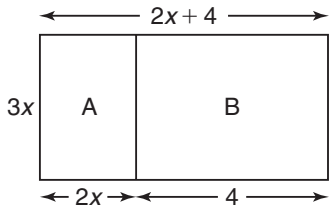
Make a rectangle with width $3x$ and length $2x + 4$.
Fill in the rectangle with tiles.



We use six x^2-tiles and twelve x-tiles.
So, $3x(2x + 4) = 6x^2 + 12x$

Use an area model

Sketch a rectangle with width $3x$ and length $2x + 4$.
The area of the rectangle is: $3x(2x + 4)$
Divide the rectangle into 2 rectangles.



Rectangle A has area: $(3x)(2x) = 6x^2$
Rectangle B has area: $(3x)(4) = 12x$
The total area is: $6x^2 + 12x$
So, $3x(2x + 4) = 6x^2 + 12x$

Use paper and pencil

$3x(2x + 4)$
Multiply each term in the brackets by the term outside the brackets.

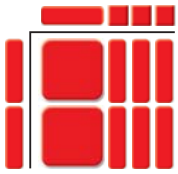
$$\begin{aligned} 3x(2x + 4) &= (3x)(2x) + (3x)(4) \\ &= 6x^2 + 12x \end{aligned}$$

Use the distributive law.

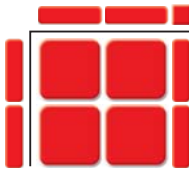
Practice

1. Write the product modelled by each set of tiles.
Determine the product.

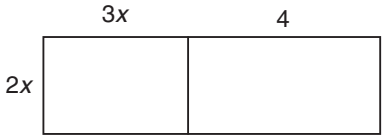
a)

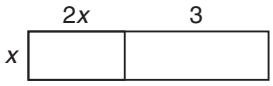

b)


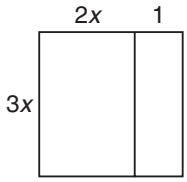
c)

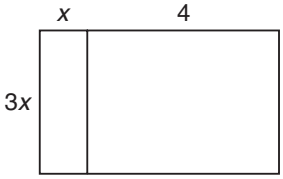

d)


2. Write the product modelled by the area of each rectangle.
Determine the product.

a)


b)


c)


d)


3. Use algebra tiles to expand.

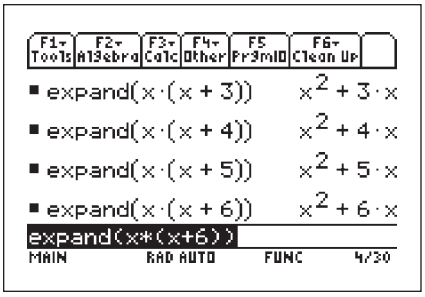
a) $x(3x + 2)$
b) $x(4x + 6)$
c) $3x(2x + 1)$
d) $4x(3x + 4)$

4. Use an area model to expand.

a) $4x(x + 3)$
b) $x(2x + 3)$
c) $2x(x + 6)$
d) $3x(5x + 2)$

5. Use the calculator screen at the right.

a) What patterns do you see?
Explain the patterns.
b) Write the next 2 lines in the patterns.



6. **Assessment Focus** Alex thinks that $x(x + 1)$ simplifies to $x^2 + 1$.
Choose a tool.
Use the tool to explain how to get the correct answer.

When we cannot use area models, we use paper and pencil.

Example

Expand: $-2x(4x^2 + 2x - 3)$

Solution Multiply each term of the polynomial by the monomial.

$$\begin{aligned} &-2x(4x^2 + 2x - 3) \\ &= (-2x)(4x^2) + (-2x)(2x) + (-2x)(-3) \\ &= -8x^3 + (-4x^2) + (+6x) \\ &= -8x^3 - 4x^2 + 6x \end{aligned}$$

7. Expand.

a) $2x(6x - 2)$ **b)** $x(8x - 2)$ **c)** $5x(x - 7)$ **d)** $x(-4x + 3)$
e) $2x(3x^2 - 4)$ **f)** $4x(-2x - 3)$ **g)** $3x(2x - 4)$ **h)** $x(-x - 1)$

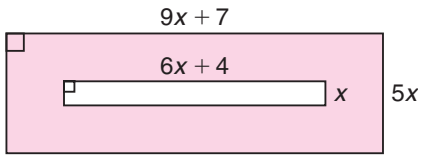
8. Expand.

a) $-2x(x + 3)$ **b)** $-x(2x + 1)$ **c)** $-x(-2x - 7)$ **d)** $-3x(2 - x)$
e) $-7x(4x - 9)$ **f)** $-x(2x + 3)$ **g)** $-2x(3x - 5)$ **h)** $-6x(2x + 3)$

9. Expand. Which method did you use each time?

a) $2x(x^2 - 2x)$ **b)** $-3x(3x^2 - 5x)$
c) $4x(8x^2 + 6)$ **d)** $7x(2x^2 + 3x - 1)$
e) $2x(-4x^2 + 5x - 7)$ **f)** $3x(4x^2 - 3x - 9)$
g) $-2x(x - 3x^2 + 1)$ **h)** $x(x^2 - 9 + x)$

10. **Take It Further**
This figure shows one rectangle inside another.
Determine an expression for the shaded area.
Simplify the expression.



In Your Own Words

How is multiplying a polynomial by a monomial like multiplying a polynomial by a constant?
How is it different?
Use examples to explain.

7.7 Solving Equations with More than One Variable Term

Investigate Using An Equation to Solve a Problem

Jackie plans to start a business selling silk-screened T-shirts.
She must purchase some equipment at a cost of $3500.
Jackie will spend $6 per shirt on materials.

Jackie's total cost, in dollars, can be represented by the expression $3500 + 6x$, where x is the number of T-shirts she makes.
Jackie sells each shirt for $11.
Her income in dollars is represented by the expression $11x$.

When Jackie's income is equal to her total cost, she breaks even.
This situation can be modelled by the equation:
$11x = 3500 + 6x$

Which tools could you use to help you?

➢ Solve the equation.
How many T-shirts does Jackie need to sell to break even?

Reflect

➢ What strategies did you use to solve the equation?

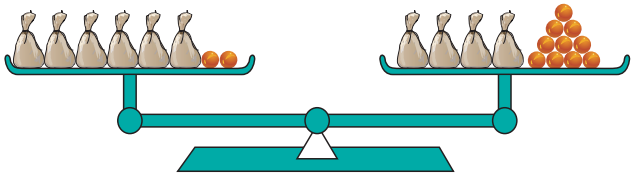
➢ If you did not use CAS, explain how you could use it to solve the equation.

➢ Suppose Jackie sold each T-shirt for $13.
What would the equation be?
What would the solution be?

Connect the Ideas

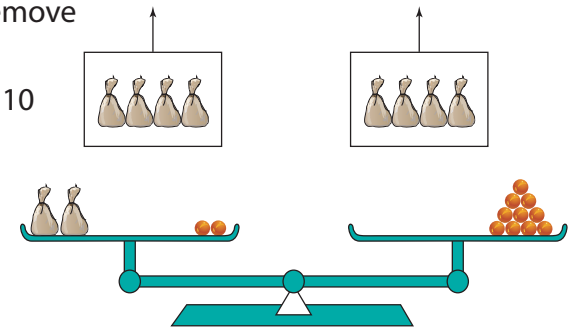
We can use balance scales to model the steps to solve an equation.

Each bag contains the same number of candies.

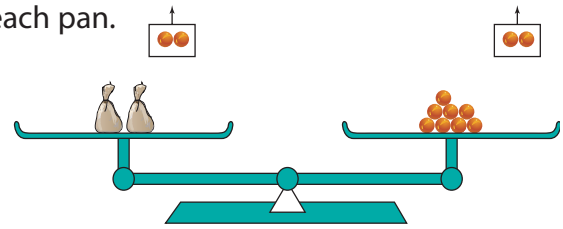


Let x represent the number of candies in a bag.
There are 6 bags and 2 candies on the left pan.
This is represented by the expression $6x + 2$.
There are 4 bags and 10 candies on the right pan.
This is represented by the expression $4x + 10$.
So, the balance scales model the equation: $6x + 2 = 4x + 10$

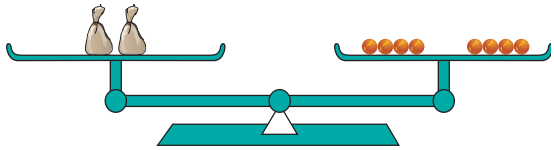
To solve the equation, remove 4 bags from each pan.
We now have: $2x + 2 = 10$



Remove 2 candies from each pan.
We now have: $2x = 8$



Divide the candies on the right pan into 2 equal groups.
Each group contains 4 candies.

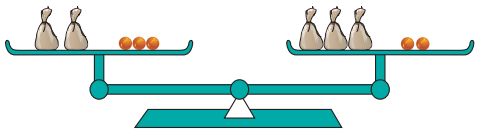


So, each bag contains 4 candies.
The solution is: $x = 4$

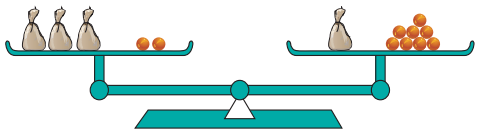
Practice

1. Write the equation represented by each balance scales.
Solve the equation.
Tell how you did it.

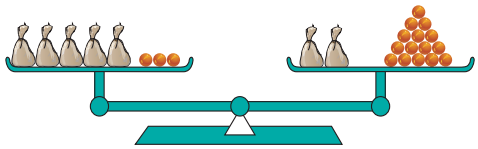
a)



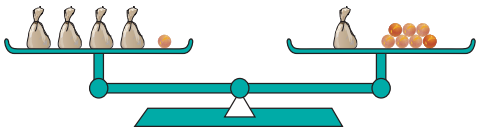
b)



c)



d)



You can use a balance strategy to solve equations with pencil and paper.

Example

Solve: $3 + x = -4x - 42$
Check the solution.

Solution $3 + x = -4x - 42$
There are variables on both sides of the equals sign.
Perform the same operations on each side until the variables are on one side of the equals sign.

When we add the opposite term, we use inverse operations.

$$3 + x = -4x - 42 \quad \text{To remove } -4x\text{, add } 4x \text{ to each side.}$$
$$3 + x + 4x = -4x - 42 + 4x \quad \text{Simplify.}$$
$$3 + 5x = -42 \quad \text{To remove 3, add } -3 \text{ to each side.}$$
$$3 + 5x - 3 = -42 - 3$$
$$5x = -45 \quad \text{To isolate } x\text{, divide by 5.}$$
$$\frac{5x}{5} = \frac{-45}{5}$$
$$x = -9$$

Check the solution.
Substitute $x = -9$ in $3 + x = -4x - 42$.

$$\begin{aligned} \text{L.S.} &= 3 + x \\ &= 3 + (-9) \\ &= 3 - 9 \\ &= -6 \end{aligned} \qquad \begin{aligned} \text{R.S.} &= -4x - 42 \\ &= -4(-9) - 42 \\ &= 36 - 42 \\ &= -6 \end{aligned}$$

The left side equals the right side.
So, the solution is correct.

2. Solve. Explain your steps.

a) $7 + x = 2x$ b) $3x + 4 = 2x$ c) $6 + 2x = x$
d) $6x - 2 = 7x$ e) $4x = 7 - 3x$ f) $3x = 2x + 8$

3. Solve.

a) $3x + 4 = 2x - 3$ b) $-5 + 9x = 11 + 5x$
c) $2 + 7x = 2x - 3$ d) $x + 12 = 30 - 2x$
e) $5 - 7x = -3x + 9$ f) $-11 + 6x = -6x + 13$
g) $-4x + 12 = 2x + 18$ h) $50 + 7x = 8x + 1$

4. Choose two of the equations you solved in question 3.
Check your solution.

5. Solve each equation.

a) $2x + 12 = x + 20$ b) $-6x + 15 = -x + 5$
c) $-12 + 18x = 3x + 3$ d) $-2x + 3x = 4x + 9$
e) $5 - 6x - 12 = 14 - 7x$ f) $4x - 5x + 7 = 2x - 14$

6. **Assessment Focus** Solve each equation.
Show your work.

a) $3x - 5 = 7 - 3x$ b) $12 + 3x = x - 14$
c) $-6x - 10 = 3x + 8$ d) $-x = x + 6$
e) $9 - 6x = x + 2$ f) $8x - 4 - 3x = 11 + 4x$

7. An auto parts manufacturer buys a machine to produce a specific part.
The machine costs \$15 000.
The cost to produce each part is \$2.
The parts will sell for \$5 each.
Let x represent the number of parts produced and sold.
To break even, the cost, in dollars, $15\,000 + 2x$ must equal the income $5x$.
This can be modelled by the equation: $15\,000 + 2x = 5x$
Solve the equation.
What does the solution represent?

8. **Take It Further** Solve each equation.

a) $4(x - 3) = 3x$ b) $12(5 - x) = 72$ c) $-3(2x - 5) = -x + 5$

In Your Own Words

How many different ways can you solve an equation?
Use an example to illustrate your answer.

Chapter Review

What Do I Need to Know?

Polynomial

A polynomial is the sum or difference of one or more terms.

Algebra Tiles

We can represent a polynomial with algebra tiles.

$-2x^2 + 4x + 3$

Monomial

A monomial is a polynomial with one term.
2, $3x$, $-4x^2$, and $5x^3$ are examples of monomials.

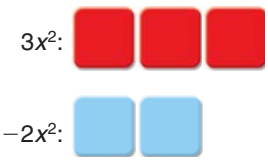
Constant Term

A constant term is a term without a variable; for example, 5.

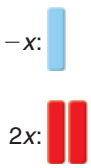
Like Terms

Like terms are represented by the same type of algebra tile.
These are like terms:

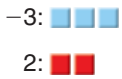
$3x^2$ and $-2x^2$



$-x$ and $2x$



-3 and 2



The Distributive Law

Each term in the brackets is multiplied by the term outside the brackets.

$$-2(-4x^2 + 3x - 2) = 8x^2 - 6x + 4$$

Solving an Equation

To solve an equation means to determine the value of the variable that makes the equation true.

What Should I Be Able to Do?

7.1 **1.** Explain why x^2 and $2x$ are not like terms.

2. Which expression does each group of tiles represent?

a) 

b) 

c) 

3. Use algebra tiles to show each expression.
Sketch the tiles you used.

a) $3x^2 + 4x - 2$
b) $-x^2 - 6x$

4. Simplify.

a) $5 + 7x + 2$
b) $4x + 6 + 3x$
c) $2x^2 - 8x + x^2 + 3x$
d) $6 + 3x - 3 - 2x$
e) $2x^3 - 3 + 6 - 5x^3$
f) $-2x^3 - x + 3x^3 + 5x$

5. Write an expression with 5 terms that has only 3 terms when simplified.

7.2 **6.** Add. Use algebra tiles.

a) $(4x + 3) + (3x - 5)$
b) $(x^2 + 6x) + (-3x^2 - 7x)$
c) $(3x^2 + 1) + (2x^2 - 3)$
d) $(x + 4) + (2x + 7)$
e) $(3x^2 - 2x + 5) + (4x^2 + 5x - 7)$
f) $(5x^2 + 7x - 3) + (-2x^2 - 3x + 2)$
g) $(x^2 - 3x + 4) + (3x^2 - 4x - 3)$

7. Add. Which tools can you use?

a) $(6x - 2) + (3x - 1)$
b) $(2x^2 + 6x) + (-3x^2 - 2x)$
c) $(3x^2 - 6x + 8) + (-5x^2 - x + 4)$
d) $(-x^2 - 2x + 8) + (4x^2 - x + 3)$

7.3 **8.** Subtract. How could you check your answers?

a) $(x - 8) - (4 + x)$
b) $(x^2 + 4) - (3x^2 - 5)$
c) $(7x^2 - 2x) - (5x^2 + 3x)$
d) $(4x^2 - 3x - 1) - (2x^2 - 5x - 3)$
e) $(-6x^2 + 5x + 1) - (2x^2 - x - 4)$
f) $(2x^2 - 7x + 9) - (x^2 - 3x - 3)$

7.4 **9.** Expand.

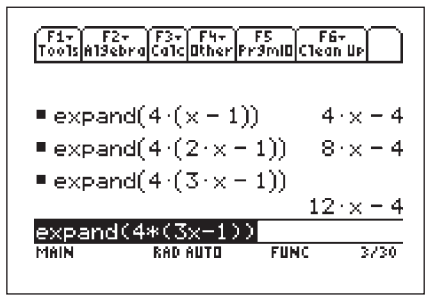
Which tools could you use to help you?

a) $5(3x^2 + 4)$
b) $4(7 + 2x)$
c) $3(x^3 - 2x^2 + 2)$
d) $8(2x^2 + 3x - 4)$
e) $7(2x^3 - 1)$

10. Expand.

a) $-6(x + 2)$
b) $-7(x^2 - 4)$
c) $-2(2x^2 + 1)$
d) $-5(3x^3 - 2x - 3)$
e) $-3(x^3 - 2x^2 + 4)$

11. Write the next 3 lines in the pattern shown on the calculator screen.



7.5 **12.** Use algebra tiles to explain why:

a) $3x + 2x$ equals $5x$

b) $(3x)(2x)$ equals $6x^2$

13. a) Simplify.

i) $(8x)(3x)$

ii) $(-2x)(-6x)$

iii) $(4x^2)(-3x)$

iv) $(-9x)(-2x^2)$

b) For which products in part a can you use algebra tiles? Explain.

7.6 **14.** Expand. For which products can you use algebra tiles? Explain.

a) $x(3x + 4)$

b) $3x(x^2 - 8)$

c) $2x(4 - x)$

d) $6x(-3x^2 + 4x + 2)$

15. Expand.

a) $-3x(-2x + 3)$

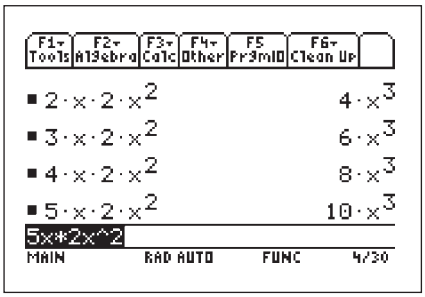
b) $-2x(x^2 - 5)$

c) $-5x(3x + 7)$

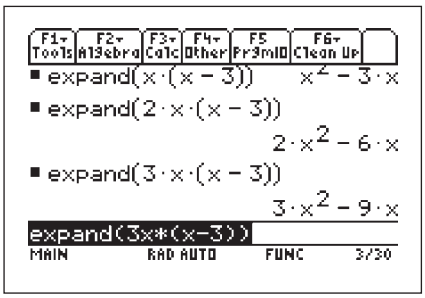
d) $2x(4x^2 + 5x - 3)$

16. Write the next 3 lines in the pattern shown on each screen.

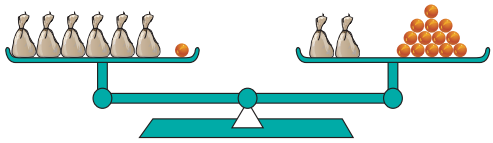
a)



b)



7.7 **17.** a) Let x represent the number of candies in each bag.
Write the equation represented by the scales.



b) Solve the equation.

c) How many candies are in each bag?

18. Solve each equation.

a) $5x + 8 = x$

b) $3x + 3 = x + 7$

c) $2x + 10 = 4$

19. Solve each equation.

a) $5x - 4 = 8 + 3x$

b) $6 + 3x = x - 2$

c) $12 + x = -2x + 9$

d) $2x - 3 = 6 - x$

20. A fund-raiser is organized for hurricane victims. With the purchase of a \$100 ticket, each person is given a souvenir bracelet (value \$20) and the chance to win a car.
Let x represent the number of tickets sold.
Then, the income, in dollars, from ticket sales is $100x$.
The expenses, in dollars, are $20\,000 + 20x$.
The organizers of the fundraiser would like to raise \$60 000 after all expenses. This can be modelled by the equation:
$100x = 60\,000 + 20\,000 + 20x$

a) Solve the equation.

b) What does the solution represent?

Practice Test

Multiple Choice: Choose the correct answer for questions 1 and 2.

1. Which polynomial is simplified?

A. $3x + 4 - x^2 + 8$ **B.** $3x^3 - 2x + x^2 - x$

C. $x^2 - 6 + x$ **D.** $x + 6x - x^2 + 7$

2. What is the solution to $80 + 10x = 30x - 20$?

A. $x = 3$ **B.** $x = 3.5$

C. $x = 5$ **D.** $x = -5$

Show your work for questions 3 to 6.

3. **Knowledge and Understanding** Simplify.

Which tools could you use to help?

a) $(3x^2 + 4x - 1) + (2x^2 - 8x - 4)$

b) $(x^2 + 3x - 2) - (2x^2 + x - 2)$

c) $3(x + 4)$

d) $(2x)(3x^2)$

e) $4x(x^2 - 5x + 3)$

4. **Application** The cost to rent a hall for the prom is \$400 for the hall and \$30 per person for the meal. This can be modelled by the equation $C = 400 + 30x$, where x is the number of students attending.

a) Suppose 150 students attend.
What will be the cost of the prom?

b) The prom committee has \$10 000.
What is the greatest number of students that can attend with this budget?

5. **Communication** How can you tell if a polynomial can be simplified?
Include examples in your explanation.

6. **Thinking** Joe subtracted $(4x^2 - 3x) - (2x^2 - 5x + 4)$. He got the answer $2x^2 - 8x + 4$.

a) What mistake did Joe likely make? Explain.

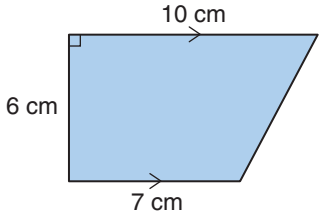
b) Determine the correct answer.

c) How could you check your answer is correct?

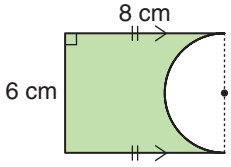
CHAPTER

1 **1.** Determine the perimeter and area of each figure. The curve is a semicircle.

a)

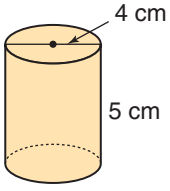


b)



2. Determine the volume of each object.

a)



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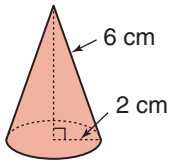
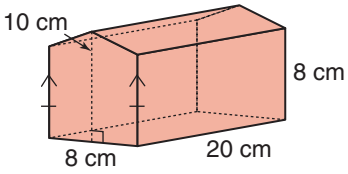
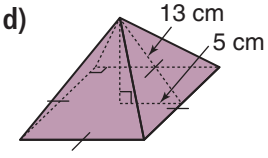


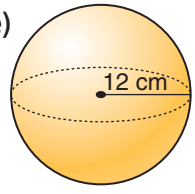
c)



d)



e)



2 **3.** a) For each perimeter, determine the dimensions of the rectangle with the maximum area.

i) 40 cm ii) 68 m iii) 90 cm

b) Calculate the area of each rectangle in part a. How do you know each area is a maximum?

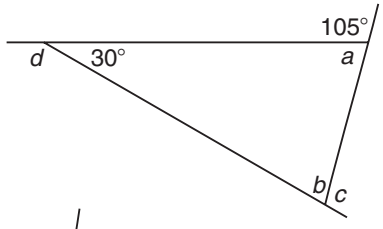
4. Determine the dimensions of a rectangle with area 144 cm^2 and the minimum perimeter. What is the minimum perimeter?

5. A patio is to be built on the side of a house using 48 congruent square stones. It will then be surrounded by edging on the 3 sides not touching the house. Which designs require the minimum amount of edging? Include a labelled sketch in your answer.

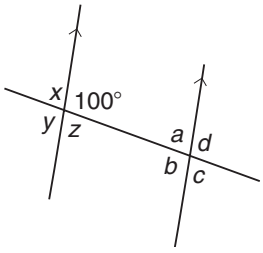
3 **6.** Can a right triangle have an obtuse angle? Explain.

7. Determine the angle measure indicated by each letter. Justify your answers.

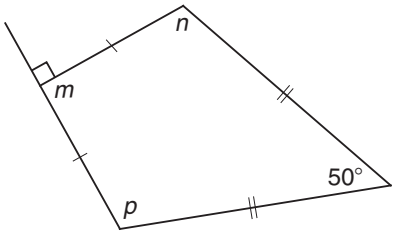
a)



b)



c)



8. Determine the measure of one exterior angle of a regular polygon with each number of sides. Show your work.

a) 5 sides b) 9 sides c) 10 sides

CHAPTER

4 **9.** Determine the value of each variable.

a) $2:5 = 6:n$ b) $3:b = 7:42$

c) $a:15 = 4:10$ d) $9:12 = 15:m$

10. Determine each unit rate.

a) 288 km driven in 4 h

b) \$3.79 for 3 kg of apples

c) \$51 earned for 6 h of work

d) 9 km hiked in 2.5 h

e) \$3.49 for 6 muffins

11. Maria is swimming lengths. She swims 2 lengths in 1.5 min. At this rate:

a) How far can she swim in 9 min?

b) How long will it take her to swim 30 lengths?

12. A bathing suit is regularly priced at \$34.99. It is on sale at 30% off.

a) What is the sale price?

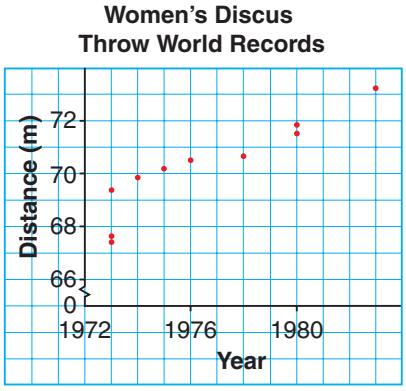
b) How much does the customer pay, including taxes?

13. Tomas borrows \$2500 for 6 months. The annual interest rate was 3%.

a) How much simple interest does Tomas pay?

b) What does the loan cost Tomas?

5 **14.** Your teacher will give you a large copy of the scatter plot below.



a) What does the scatter plot show?

b) What was the world record in women's discus in 1975? In 1980?

c) In what year was the world record about 70 m?

d) Draw a line of best fit.

e) Estimate the world record in 1984. Explain how you did this.

15. Here are the dimensions of some rectangles with perimeter 30 cm.

Width (cm)	1	2	3	4	5	6	7
Length (cm)	14	13	12	11	10	9	8

a) Is the relationship between length and width linear? Justify your answer.

b) Graph the data. Does the graph illustrate your answer to part a? Explain.

c) Use the graph.

i) Determine the length when the width is 5.5 cm.

ii) Determine the width when the length is 13.5 cm.

d) Write a rule for the relationship.

16. A baseball is thrown up into the air. Its height is measured every 0.2 s.

Time (s)	0	0.2	0.4	0.6	0.8	1.0	1.2
Height (m)	1.0	2.1	2.8	3.0	2.9	2.5	1.6

a) Graph the data.

b) Draw a curve or line of best fit.

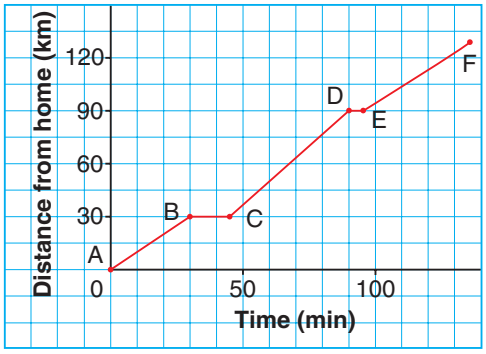
c) What is the greatest height the ball reaches?

d) When will the ball hit the ground?

e) At what times is the ball 2 m above the ground?

CHAPTER

17. The graph shows Tyler's distance from his home as he drives to his cottage.



a) Describe Tyler's drive.

b) Tyler makes 2 stops: one to buy gas and another to pick up his cousin. Which part of the graph do you think represents each stop? Justify your answers.

6 **18.** This pattern is made of toothpicks. It continues.

a) Sketch the next frame in the pattern.

b) Copy and complete the table below for the first 6 frames. Is the relation linear or non-linear? Explain.

Frame number	Number of toothpicks	First differences
1		
2		
3		

c) Graph the relationship. Does the graph support your answer to part b? Explain.

19. The table shows the value in Philippine pesos of different amounts in Canadian dollars in early 2006.

Amount in dollars	10	55	48	20	36
Amount in pesos	450	2475	2160	900	1620

a) Graph the data. Does the graph represent direct variation? Explain.

b) Determine the rate of change. Explain what it represents.

c) Write an equation for the value, p pesos, of d dollars.

d) Determine the value of \$175.

e) Determine the value of 7500 pesos.

f) How did you answer parts d and e? Did you use the table, graph, or equation? Explain.

20. Refer to question 19. In early 1999, \$1 was worth about 25 Philippine pesos.

a) How would the graph change? Draw the new graph on the grid in question 19.

b) How would the equation change? Write the new equation.

21. Sunil is joining a tennis club for the summer. The club offers a special 10-week summer membership for students. It costs \$50, plus \$3.50 per hour of court time.

a) Make a table. Show the total costs for times played from 0 h to 60 h.

b) Graph the data. Does the graph represent direct variation? Explain.

c) Write an equation to determine the total cost, C dollars, when n hours of tennis are played.

d) Suppose Sunil has budgeted \$200 for tennis costs. How many hours can he play? How did you determine your answer?

22. Refer to question 21. The tennis club also offers a special 10-week summer membership for adults. It costs \$75, plus \$4.00 per hour of court time.

a) How would the graph change? Draw the new graph on the grid in question 21.

b) How would the total cost equation change? Write the new equation.

c) How many hours could an adult play if her budget for tennis is \$200?

23. The daily cost of running a sausage cart is a fixed cost of \$40, plus \$1 per sausage. The revenue is \$3 per sausage.

a) Write an equation for the daily cost, C dollars, in terms of n, the number of sausages sold.

b) Write an equation for the revenue, R dollars, in terms of n.

c) Graph the equations on the same grid.

d) How many sausages have to be sold before a profit is made? Explain.

24. What expression does each group of algebra tiles represent?

a)

b)

25. Simplify each expression by combining like terms.

a) $8x + 3 - x$

b) $3x^2 - 7x - x^2 + 4x$

c) $10 + 2x - 5 - 2x$

d) $3x^3 + 6x + 2x^3 - 2x$

26. Add. Which tools can you use?

a) $(5x + 3) + (6x - 7)$

b) $(2x^2 - 9x) + (4x - 3x^2)$

c) $(-x^2 + 3x - 1) + (3x^2 - 7x + 2)$

27. Subtract. How could you check your answers?

a) $(2x + 5) - (x + 2)$

b) $(7x^2 - 8) - (6x^2 - 3)$

c) $(5x^2 + x - 2) - (-2x^2 + 3x - 3)$

28. Expand.

a) $6(5 + 2x)$

b) $2(x^3 - 3x^2 + 3)$

c) $-3(3x + 1)$

d) $-4(2x^3 - 5x^2)$

e) $x(2x + 7)$

f) $3x(7 - 3x)$

g) $-2x(2x^2 - 3)$

h) $2x(3x^2 - 4x + 5)$

29. Solve each equation. How could you check your answer?

a) $7x + 6 = 10 + 3x$

b) $8 + 5x = x - 4$

c) $9 - 3x = -5x + 3$

d) $2x - 2 = 4 - x$