6 Linear Relations

What You'll Learn

How linear relations connect to earlier work in this text on proportional reasoning, and geometry and measurement concepts

And Why

Real-life situations can be represented as linear relations in different ways. This helps to show how math relates to life outside school.

Key Words

- first differences
- rise
- run
- rate of change
- direct variation
- partial variation
- vertical intercept

Project Link

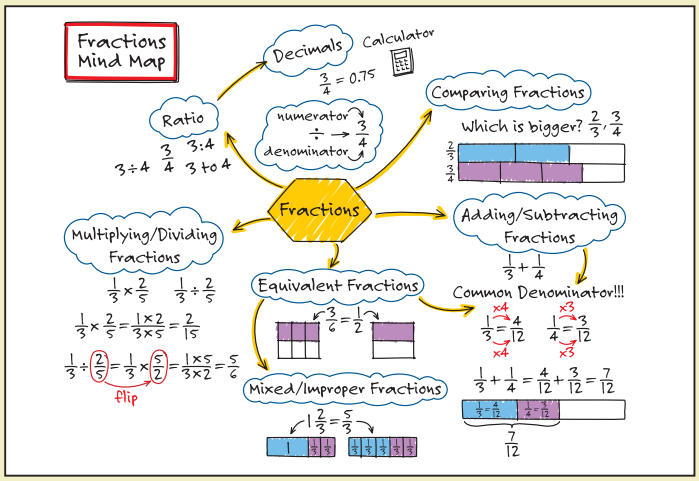
- Heart Rates, Breathing Rates, and Exercise

Reading and Writing in Math

Making a Mind Map

Organizing information can help you learn and remember the important ideas.
A *mind map* is one way to do this.

Here is a student's mind map on fractions.



To make your own mind map:

- In the centre of a piece of paper, write the chapter title: Linear Relations
- As you work through this chapter, list all the important ideas. Make sure to include:
  - section titles, such as *Direct variation* and *Partial variation*
  - new words
  - key ideas
- At the end of each section, compare your list with a classmate's list. Arrange your ideas on your map.
  Look for connections among the ideas.
  Organize your map to highlight these relationships.
- At the end of the chapter, make a final copy of your map.
  Use colour and diagrams to highlight and explain important concepts. Use arrows to link ideas.
- Compare your map with those of your classmates.
  Talk with a classmate.
  - How could you have organized the information differently?
  - What was effective about each map?
  - How could you improve your mind map?

6.1 Recognizing Linear Relations

Investigate Using a Table of Values to Explore a Relationship

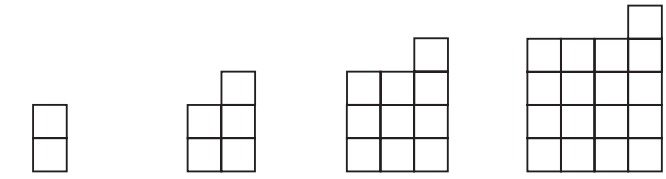
Work in a group of 4.

➢ You will need linking cubes.
Place a cube on the desk.
Five faces of the cube are visible.
Join 2 cubes. Place them on the desk.
Eight faces of the cubes are visible.
Record the data in a table.
Continue the table to show data for up to 10 cubes in a line.

Number of cubes	Number of faces visible

What patterns do you see in the table?
Graph the data. How are the patterns shown in the graph?
Is the relationship linear or non-linear? How do you know?

➢ Consider this pattern of cubes.



Use linking cubes to build these frames.
How many cubes are in each frame?
Record your data in a table.
Continue the table to show data for 6 frames.

Frame number	Number of cubes

What patterns do you see in the table?
Graph the data. How are the patterns shown in the graph?
Is the relationship linear or non-linear? How do you know?

Reflect

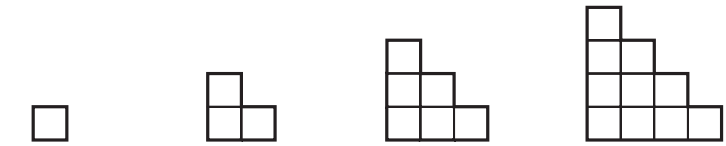
Suppose you have a table of values.
How can you tell if the relationship it represents is linear or non-linear without drawing a graph?

Connect the Ideas

Look at a pattern

Use square tiles, grid paper, or square dot paper to help you picture the pattern.

Here is a pattern of squares. The pattern continues.



Record the data

Here are tables for the perimeter and area of each frame.

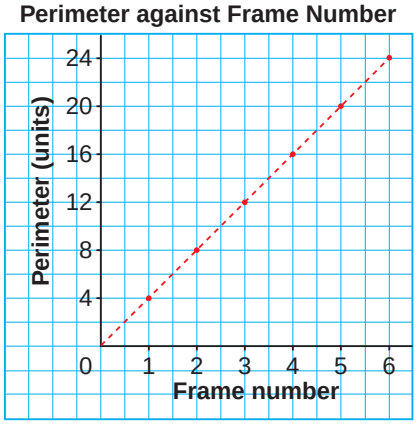
Notice that the frame numbers increase by 1 each time.

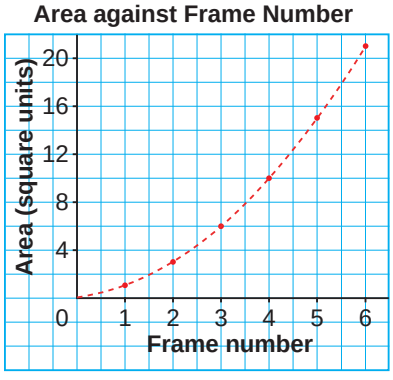
Frame number	Perimeter (units)
1	4
2	8
3	12
4	16
5	20
6	24

Frame number	Area (square units)
1	1
2	3
3	6
4	10
5	15
6	21

Graph the data

Here are the graphs of the data.





Analyse the data

The graph of *Perimeter* against *Frame number* shows a linear relation.
The points lie on a straight line.
We join the points with a broken line to show the trend.

The graph of *Area* against *Frame number* shows a non-linear relation.
The points lie along a curve.
We join the points with a broken curve to show the trend.

The broken line and broken curve indicate that only plotted points are data points.

First differences for a linear relation

Add a third column to the Perimeter table.
Record the changes in perimeter.

Frame number	Perimeter (units)	Change in perimeter (units)
1	4	
2	8	$8 - 4 = 4$
3	12	$12 - 8 = 4$
4	16	$16 - 12 = 4$
5	20	$20 - 16 = 4$
6	24	$24 - 20 = 4$

The numbers in the third column are called **first differences**.
The first differences for a linear relation are equal.
For every increase of 1 in the frame number,
the perimeter increases by 4 units.

All linear relations have first differences that are constant.
We can use this to identify a linear relation from its table of values.

First differences for a non-linear relation

Add a third column to the Area table.
Record the changes in area.

Frame number	Area (square units)	Change in area (square units)
1	1	
2	3	$3 - 1 = 2$
3	6	$6 - 3 = 3$
4	10	$10 - 6 = 4$
5	15	$15 - 10 = 5$
6	21	$21 - 15 = 6$

For a non-linear relation, the first differences are not equal.
For every increase of 1 in the frame number,
the area increases by a different amount each time.

Practice

1. For each table below, determine the first differences.
State whether the data represent a linear or non-linear relation.
Explain how you know.

a)

Time rented (h)	Cost ($)
0	0
1	6
2	12
3	18
4	24

b)

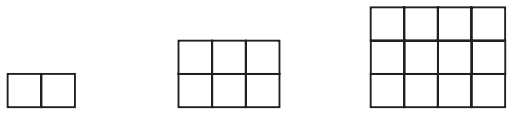
Age (months)	Mass (kg)
0	3.0
1	4.0
2	4.8
3	5.5
4	6.2

c)

Time (s)	Distance (m)
0	15
1	20
2	25
3	30
4	35

2. Suggest a possible situation that each table in question 1 could represent.

3. This pattern was constructed using square tiles:



Frame 1 Frame 2 Frame 3

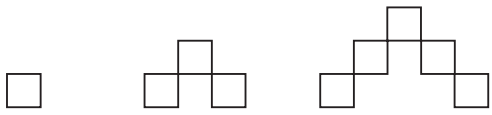
a) Copy and complete this table.
Record data for the first 6 frames.

Frame number	Area (square units)

b) Determine the first differences.
Is the relationship between frame number and area linear? Explain.

c) Graph *Area* against *Frame number*.
Does the graph support your answer to part b? Explain.

4. This pattern was constructed using square tiles:



Frame 1 Frame 2 Frame 3

a) Copy and complete this table.
Record data for the first 6 frames.

Frame number	Number of tiles

b) Determine the first differences.
Is the relationship between frame number and number of tiles linear? Explain.

c) Graph *Number of tiles* against *Frame number*.
Does the graph support your answer to part b? Explain.

d) How many tiles will be in the 9th frame?
How did you find out?

First differences can be negative.

Example

This table shows the money in a bank account when it was opened, and at the end of each following week.

Number of weeks	Account balance ($)
0	200
1	175
2	150
3	125
4	100

a) Determine the first differences. What do the first differences represent?

b) Is this a linear or non-linear relation? Explain.

c) Graph this relationship. Describe how the patterns in the table are shown in the graph.

d) How much money is in the account after 6 weeks?

Solution

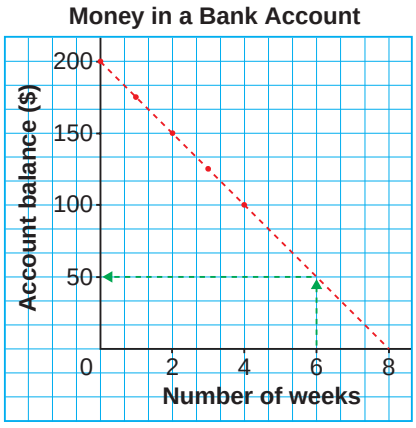
a)

Number of weeks	Account balance ($)	First differences
0	200	
1	175	$175 - 200 = -25$
2	150	$150 - 175 = -25$
3	125	$125 - 150 = -25$
4	100	$100 - 125 = -25$

The first differences represent the change in the account balance each week. Since the first differences are negative, money is being withdrawn from the account.

b) The first differences are equal. The relationship is linear.

c)



The points lie on a straight line that goes down to the right. For every unit you move to the right, you move 25 units down.

d) From the graph in part c, there are $50 in the account after 6 weeks.

5. A barrel contained 42 L of water. The water was leaking out. The table shows how the volume of water in the barrel changed every hour.

Time (h)	Volume (L)
0	42
1	38
2	34
3	30
4	26

a) Determine the first differences. What do the first differences represent?
b) Is the relationship linear or non-linear? Explain.
c) Graph the relation. Does the graph support your answer to part b? Explain.
d) How much water would be in the barrel after 6 h? What assumptions did you make?
e) Write a question you could answer using the data. Answer the question.

6. **Assessment Focus** The table shows the areas of rectangles for which the length is 3 times the width.

Width (cm)	Area (cm^2)
1	3
2	12
3	27
4	48

a) Sketch the rectangles.
b) Determine the first differences.
c) Is the relationship linear or non-linear? Explain.
d) Graph the relationship. Describe the graph.
e) Predict the area of the next rectangle in the pattern. Sketch the rectangle. Calculate its area to check your prediction.

7. Laura is swimming lengths to prepare for a triathlon. The table shows the distances she swims in metres.

Number of lengths	Distance (m)
0	0
5	125
10	250
15	375
20	500

a) By how much are the numbers in the first column increasing?
b) Determine the first differences. What do the first differences represent?
c) Is the relationship linear or non-linear? Explain.
d) Graph the relationship. Does the graph support your answer to part c? Explain.
e) What is the length of the pool? How did you find out?

8. **Take It Further** Make your own pattern using square tiles, grid paper, or square dot paper. Choose 2 properties of the figures in your pattern; such as frame number, height, side length, perimeter, area, or number of squares. Investigate whether the number pattern that relates the properties is linear or non-linear. Show your work.

In Your Own Words

Describe two ways to identify whether a relationship is linear or non-linear. Give an example of each way.

6.2 Average Speed as Rate of Change

Investigate Comparing Average Speeds on a Graph

Work in a group of 4.
You will need a stopwatch or a watch with a second hand and grid paper.

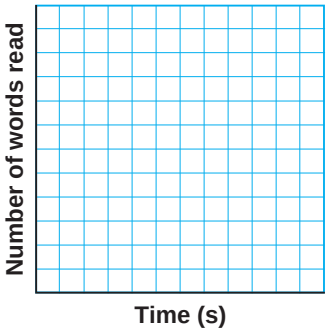
Take turns to read this tongue twister aloud 5 times.

A tongue twister is a phrase or sentence that is hard to say fast because the words include similar sounds.

"A big bug bit the little beetle but the little beetle bit the big bug back."

Try to say it as quickly as possible without stumbling.
Record the time in seconds it takes.
The tongue twister contains 16 words.
So, when you read it 5 times, you are reading 5×16, or 80 words.
Copy and complete this table:

Reader's name	Time (s)	Number of words read
		80
		80
		80
		80



Plot *Number of words read* against *Time* for each reader.
Use a ruler to draw a line from (0, 0) to each plotted point.
Label each line with the reader's name.

Who read the fastest? How do you know?
Order the readers from the fastest to the slowest.
Order the lines on the graph from steepest to least steep.
Compare the order with that of the readers. What do you notice?

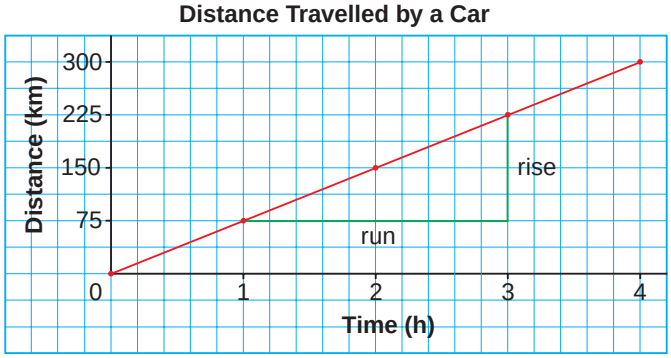
Reflect

How do you think your average reading speed for a tongue twister compares with that for regular text?
How could you check your prediction?

Connect the Ideas

This graph shows how the distance changes over time during a trip.

This is called a distance–time graph.



The graph is linear.

Recall that: Average speed $= \frac{\text{Distance travelled}}{\text{Time taken}}$

Choose any two points on the line, such as (1, 75) and (3, 225).

The distance travelled is the vertical distance between the points on the graph, or the **rise**.

The rise is: 225 km − 75 km = 150 km

The time taken is the horizontal distance between the points, or the **run**.

The run is: 3 h − 1 h = 2 h

$$\begin{aligned}\text{Average speed} &= \frac{\text{Distance travelled}}{\text{Time taken}}\\ &= \frac{\text{rise}}{\text{run}}\\ &= \frac{150\text{ km}}{2\text{ h}}\\ &= 75\text{ km/h}\end{aligned}$$

The average speed is 75 km/h.

$\frac{\text{rise}}{\text{run}}$ is called the **rate of change**.

It is a measure of the steepness of the line.

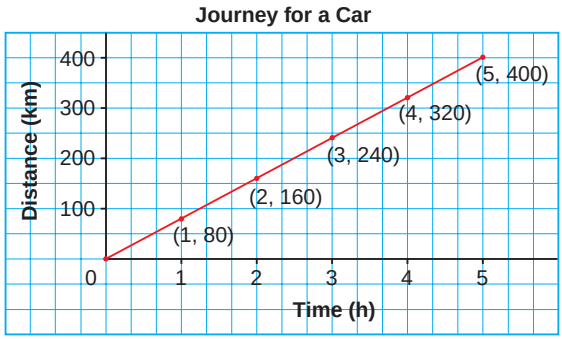
The rate of change tells you how many units to move up or down for every unit you move to the right on the graph.

The *rate of change* of distance over time is the average speed.

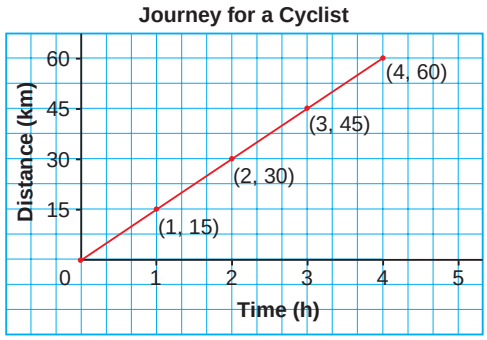
Practice

1. Determine each rate of change.

a)



b)



2. What does each rate of change in question 1 represent?

3. In *Connect the Ideas*, page 198, the rate of change was calculated using 2 points on the line. Choose 2 different points on the line. Confirm that the rate of change is the same for these points.

An average speed can be expressed in different units.

Example

Nathalie leaves downtown Ottawa to meet a friend at Dow's Lake, 6 km away. She skates along the Rideau Canal.
It takes Nathalie 5 min to skate 1 km.

a) Assume Nathalie keeps skating at this rate.
Make a table to show her distance from downtown at 5-min intervals.
Graph the data.

b) Determine the rate of change from the graph.
What does the rate of change represent?

c) How long does it take Nathalie to reach Dow's Lake?
How did you determine this?

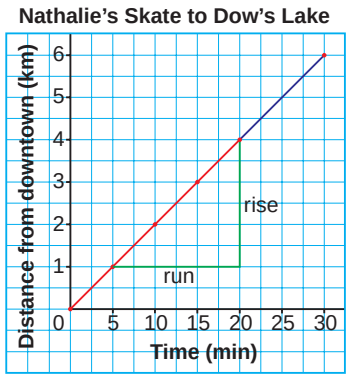
d) What is Nathalie's average speed in kilometres per hour?

Ottawa's Rideau Canal Skateway was officially designated the world's largest naturally frozen ice rink.

Solution

a)

Time (min)	Distance from downtown (km)
0	0
5	1
10	2
15	3
20	4



b) Choose 2 points on the line: (5, 1) and (20, 4)
The rise is: 4 km − 1 km = 3 km
The run is: 20 min − 5 min = 15 min

$$\text{Rate of change} = \frac{\text{rise}}{\text{run}} = \frac{3\text{ km}}{15\text{ min}} = 0.2\text{ km/min}$$

The rate of change is 0.2 km/min.
This is Nathalie's average speed in kilometres per minute.

c) By extending the graph, we can see that it takes Nathalie 30 min to travel the 6 km to Dow's Lake.

d) Nathalie's average speed is 0.2 km/min.
There are 60 min in 1 h.
So, the average speed in kilometres per hour is: $60 \times 0.2 = 12$
Nathalie's average speed is 12 km/h.

To answer the questions in parts c and d, we could have extended the table.

4. Matt took 20 min to drive 24 km to attend a concert.
- **a)** Graph distance against time for this journey.
- **b)** What is the rate of change of distance with time?
- **c)** What was Matt's average speed for his trip? How do you know?
- **d)** What was Matt's average speed in kilometres per hour?

Need Help?
Read the Example above.

5. Assessment Focus A group of friends is hiking 15 km around a lake.
It takes the group 30 min to hike the first 2 km.
Assume the group keeps hiking at this rate.
- **a)** Make a table to show the distance from the start at 30-min intervals. Graph the data.
- **b)** Determine the rate of change. What does it represent?
- **c)** How long will it take for the group to complete the hike? How did you determine this?
- **d)** What is the average hiking speed in kilometres per hour?
- **e)** Write a question you could answer using the graph. Answer the question.

6. Take It Further Keta and Aaron run in a half-marathon.
Keta can run the 21-km course in 120 min.
Aaron's average speed is 8.4 km/h.
Who runs faster? How do you know?
How many different ways can you find out?

In Your Own Words

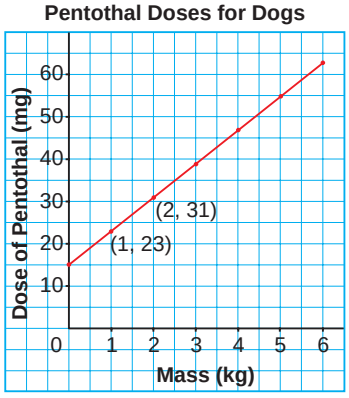
What is the rate of change? How is it related to the steepness of a line?
Make a Frayer model to explain.

6.3 Other Rates of Change

Before an operation, an animal must be "put out" so that it does not feel any pain. Many veterinarians use a drug called Pentothal.

Investigate Using Tables and Graphs to Explore Rates of Change

The graph shows the amount of Pentothal required for dogs of different masses.



Mass (kg)	Dose of Pentothal (mg)
0	
1	
2	

Record the data from the graph in a table.
Include masses up to 6 kg.

Add a third column to the table to show the first differences in the drug dose.

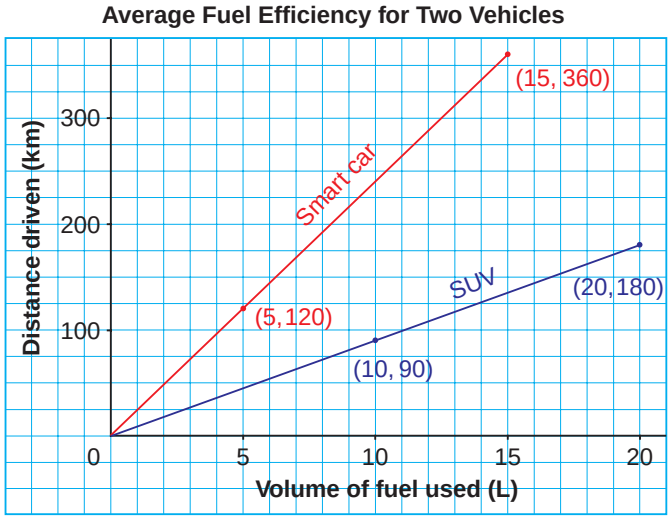
- What do you notice about the first differences? Explain.
- Select 2 points on the graph. Determine the rate of change. What does this rate of change represent?
- Compare the first differences and the rate of change. What do you notice?

Reflect

- Is this a linear relation? How do you know?
- Explain how you could determine the drug dose for any mass. What assumptions did you make?

Connect the Ideas

The graph shows the average fuel efficiency for an older sport utility vehicle (SUV) and a new Smart car.



The rate of change can be found for each vehicle.

For the Smart car:
Choose any two points on the line: (5, 120) and (15, 360)
The rise is: 360 km − 120 km = 240 km
The run is: 15 L − 5 L = 10 L

$$\text{Rate of change} = \frac{\text{rise}}{\text{run}} = \frac{240\text{ km}}{10\text{ L}} = 24\text{ km/L}$$

The Smart car drives 24 km for every litre of fuel it uses.

For the sport utility vehicle:
Choose any two points on the line: (10, 90) and (20, 180)
The rise is: 180 km − 90 km = 90 km
The run is: 20 L − 10 L = 10 L

$$\text{Rate of change} = \frac{\text{rise}}{\text{run}} = \frac{90\text{ km}}{10\text{ L}} = 9\text{ km/L}$$

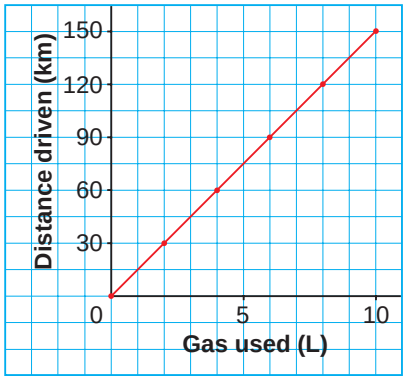
The sport utility vehicle drives 9 km for every litre of fuel it uses.

Both rates of change are positive.
The more efficient car has a greater rate of change and a steeper line on the graph. It travels farther for each litre used.

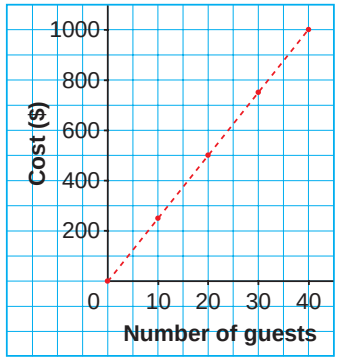
Practice

1. Determine each rate of change.

a) Gas Consumption for a Car



b) Catering Costs for Dinner Guests



2. What does each rate of change in question 1 represent?

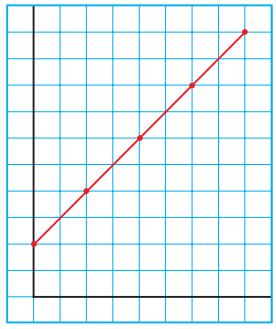
3. Use the data in *Connect the Ideas*, page 202.
The Smart car's gas tank holds 33 L of fuel.
The sport utility vehicle's gas tank holds 80 L.
Determine how far each vehicle could travel on a full tank of gas.

4. Sami conducted an experiment.
He measured the length of a spring when different masses were placed on one end of it.
Sami then calculated how much the spring stretched each time.
These are his results:

Mass (g)	0	5	10	15	20	25
Stretch (cm)	0	2	4	6	8	10

a) Graph *Stretch* against *Mass*.
b) Does the graph represent a linear relation? Explain.
c) Determine the rate of change of the stretch of the spring with mass.
What does the rate of change represent?

5. **Assessment Focus** Copy this graph on grid paper.



a) Name two quantities that could be represented by this graph.
b) Label each axis.
Give the graph a title.
c) Determine the rate of change.
Explain what it means.

A rate of change can be negative.

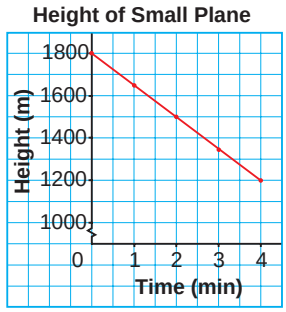
Example

The graph shows how the height of a small plane changes with time.

a) What is the plane's initial height?

b) Determine the rate of change.

c) What does the rate of change tell you about the plane's flight?



Solution

a) The plane's initial height is 1800 m.
This is the height when the time is 0.

b) Choose any two points on the line:
(0, 1800) and (4, 1200)
The rise is: 1200 m − 1800 m = −600 m
The run is: 4 min − 0 min = 4 min

$$\frac{\text{rise}}{\text{run}} = \frac{-600 \text{ m}}{4 \text{ min}}$$
$$= -150 \text{ m/min}$$

The rate of change is −150 m/min.

c) The height is decreasing at an average rate of 150 m per minute.
The plane is descending.

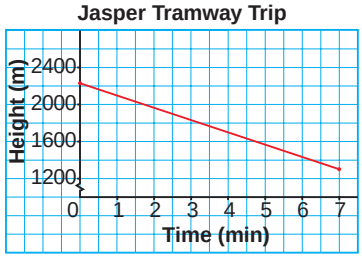
6. Suppose the plane in the *Guided Example* continues to descend at the same rate. How many minutes will it take until the plane lands? Explain your answer.

7. The Jasper Tramway is a cable car in the Canadian Rockies. The graph shows how the cable car's height changes on a 7-min trip.

a) What are the initial and final heights, to the nearest 100 m?

b) Determine the approximate rate of change.

c) What does the rate of change tell you about the cable car's trip?

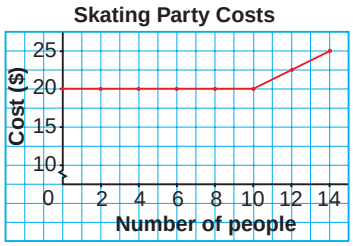


8. **Take It Further** The graph shows the costs of a skating party.

a) Determine the rate of change for each portion of the graph.

b) What do the different rates of change tell you about the costs?

c) Suppose you have a skating party for 8 people. How much will you pay for each person?



In Your Own Words

Give an example of a rate of change that is not an average speed. Describe what the rate of change measures. Include a graph.

6.4 Direct Variation

Local councils send out leaflets with their water bills.
These leaflets describe how water is wasted.

Investigate Interpreting a Graph that Passes Through the Origin

A faucet leaks. The volume of water wasted and the times when the volume was measured are shown.

- What patterns do you see in the table?
- Graph the data. Plot *Volume of water wasted* against *Time*. Describe the graph.
- Choose two points on the graph. Determine the rate of change.
- Repeat the rate of change calculation using two different points on the line.
  What do you notice?
  Explain what the rate of change represents.

Time (s)	Volume of water wasted (mL)
0	0
10	15
20	30
30	45
40	60
50	75

- Determine the first differences.
  How are they related to the rate of change?
  How are the first differences related to the change in times?
- How could you determine the volume of water wasted after each time: 100 s? 15 s? 1 s? 22 s?
- Suppose you know the time since measuring began.
  How can you determine the volume of water wasted?
  Write a rule to determine this volume.

Reflect

- Suppose you know the volume of water wasted in 20 min. How could you determine the volume wasted in 40 min?
- About how much water would be wasted by the leaking faucet in one day? Explain how you know.

Connect the Ideas

Rhonda bought sesame snacks at the bulk food store.
The cost was $1.10 per 100 g.
Here is a table of costs for different masses.

When the mass of sesame snacks doubles, the cost doubles.
When the mass of sesame snacks halves, the cost halves.

× 2 ÷ 2

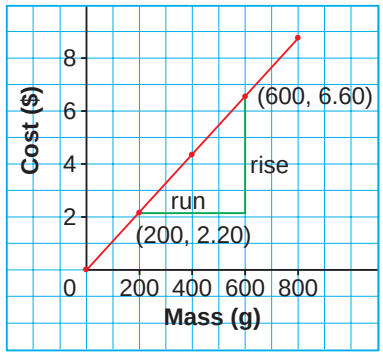
Mass of sesame snacks (g)	200	400	600	800
Cost ($)	2.20	4.40	6.60	8.80

× 2 ÷ 2

Graph the data

Recall the work in Chapter 4 on proportional reasoning. The cost of sesame snacks is directly proportional to their mass.

The graph is a straight line that passes through the origin (0, 0).
This illustrates **direct variation**.
We say that the cost *varies directly* as the mass.
When two quantities vary directly, they are *proportional*.
The cost is *directly proportional* to the mass.



Determine the rate of change

Choose any two points on the graph: (200, 2.20) and (600, 6.60)
The rise is: $6.60 − $2.20 = $4.40
The run is: 600 g − 200 g = 400 g

$$\frac{\text{rise}}{\text{run}} = \frac{\$4.40}{400\text{ g}}$$
$$= \$0.011/\text{g}$$

The rate of change is $0.011/g.
The rate of change is the unit price of sesame snacks.

Write a rule

Cost of sesame snacks in $ = (0.011) × (mass of snacks)

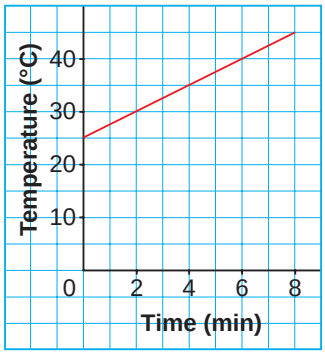
the rate of change in dollars per gram — the mass in grams

We can use the rule to determine how much Rhonda has to pay for 350 g of sesame snacks.
Substitute 350 for the mass of snacks.
Cost of sesame snacks in $ = 0.011 × 350
= 3.85
The cost of 350 g of sesame snacks is $3.85.

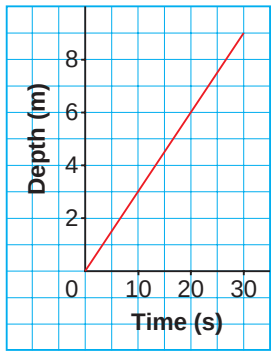
Practice

1. Does each graph represent direct variation? Explain how you know.

a) Heating of a Chemical

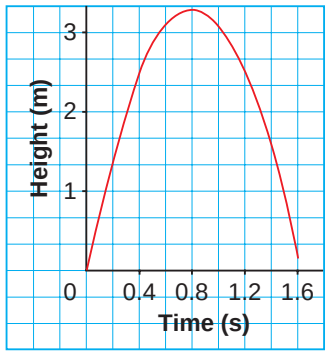


b) Scuba Diver's Depth



c) Reid's Earnings

d) Height of a Ball



2. The perimeter of a square varies directly as its side length.

Side length (cm)	Perimeter (cm)

a) Complete a table to show the perimeters of squares with side lengths from 0 cm and 5 cm.

b) Graph the data. Determine the rate of change.

c) Write a rule to determine the perimeter when you know the side length.

d) What is the perimeter when the side length is 11 cm?

3. About one-third of all water used in homes is for toilet flushes.
The table shows water use for a standard toilet manufactured after 1985.

Number of flushes	5	10	15	20	25	30
Water use (L)	65	130	195	260	325	390

a) Graph the data.
Is this a direct variation situation? Explain.

b) Determine the rate of change.
What does the rate of change represent?

c) Write a rule to determine the water use when you know the number of flushes.

d) How much water is used for 17 flushes?

4. "Nature's Best" sells bird seed for $0.86 per kilogram.

Mass of bird seed (kg)	Cost ($)

a) Copy and complete this table for masses from 1 kg to 6 kg.

b) Graph the data.
Does the graph represent direct variation? Explain.

c) Write a rule to determine the cost when you know the mass of seed.

d) How much would it cost to buy 13 kg of seed?

e) Will the cost for 26 kg be twice the cost for 13 kg? Explain your thinking.
To check your answer, use your rule to determine the cost for 26 kg.

You can use an equation to calculate values.

Example

Gas for a car is sold by the litre.
Here are the costs of gas for 5 customers at a gas station.

Volume of gas (L)	Cost ($)
10	8.50
16	13.60
6	5.10
24	20.40
18	15.30

a) Graph the data.
Does the graph represent direct variation? Explain.

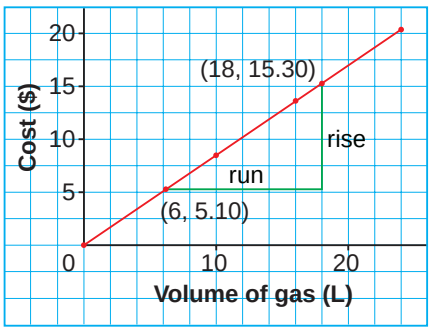
b) Determine the rate of change.
Explain what it represents.

c) Write an equation for this relation.

d) Use the equation to determine the cost of 25.8 L of gas.

Solution

a)



Yes, this graph represents direct variation.
The graph is linear and passes through the origin.

b) From the graph

The rise is: $15.30 − $5.10 = $10.20

The run is: 18 L − 6 L = 12 L

The rate of change is: $\frac{\text{rise}}{\text{run}} = \frac{\$10.20}{12\text{ L}}$
$= \$0.85/\text{L}$

The rate of change is the change in price for each litre of gas.
So, the rate of change is the unit cost of gas; that is, the cost for 1 L.

c) The cost of buying gas can be described by the rule:
Cost of gas in \$ = (unit price in \$/litre) × (volume of gas in litres)
Let C represent the cost in dollars.
The unit price is \$0.85/L.
Let v represent the volume of gas in litres.
Then an equation is: $C = 0.85 \times v$
or, $C = 0.85v$

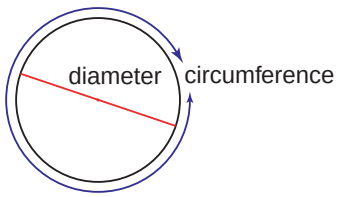
d) Use the equation: $C = 0.85v$
To determine the cost of 25.8 L of gas, substitute: $v = 25.8$
$C = 0.85 \times 25.8$
$= 21.93$
25.8 L of gas cost \$21.93.

5. Kaitlin works planting trees in Northern Ontario. She is paid 16¢ for each tree she plants.

Number of trees, n	Earnings, E (\$)
0	0
100	

a) Complete a table to show how much Kaitlin would earn for planting up to 1000 trees.
b) Graph Kaitlin's earnings.
Does the graph represent direct variation? Explain.
c) What is the rate of change?
What does it represent?
d) Write an equation for this relationship.
e) An experienced planter can plant between 1000 and 3000 trees a day.
Use the equation to determine how much Kaitlin would earn when she plants 1700 trees in one day.

6. Jamal measured several circles.
He used thread to measure the circumference and a ruler to measure the diameter.
Here are his results.



Diameter, d (cm)	Circumference, C (cm)
0	0
12	37.2
2	6.2
7	21.7
5	15.5

a) Graph the data.
Does the graph represent direct variation? Explain.
b) Determine the rate of change.
Explain what it represents.
c) Write an equation for this relationship.
d) Use the equation to estimate the circumference of a circle with diameter 13 cm.

7. Drew filled three bags of trail mix at the bulk store.

a) Graph *Cost* against *Mass* for the trail mix.
Does the graph represent direct variation? Explain.
b) Determine the rate of change. Explain what it represents.
c) Write an equation for this relation.
Use m for the mass of trail mix in grams and C for the cost in dollars.
d) Determine the cost of 340 g of trail mix. How did you do this?
e) Write a question you could answer using the graph or equation.
Answer the question.

8. **Assessment Focus** Look back at the situations and the equations that represent them. Use these to help you with this question.
Describe a situation that could be modelled by each equation.
a) $C = 0.90v$ **b)** $d = 90t$ **c)** $C = 5.50n$

9. Find a flyer from a grocery store or bulk food store that lists prices per 100 g.
Choose an item from the flyer.
Write a question about the item.
Follow the style of question 7.
Prepare a sample solution for your question.
Exchange questions with a classmate.
Answer the question you receive.
Check each other's work.

10. **Take It Further** Anastasia drives 55 km to work.
One day, she left home at 7:00 a.m. and arrived at work at 8:15 a.m.
a) What would a graph of distance against time look like for Anastasia's drive to work?
Do you have enough information? Explain.
b) What is Anastasia's average speed? Explain.

In Your Own Words

How do you know when a graph represents direct variation?
Explain with examples of graphs and equations.

6.5 Partial Variation

Carlo works for a car-sharing co-operative.
The co-op owns a few cars.
Co-op members can arrange to use the cars by the hour, day, or weekend.

Investigate A Linear Relation Involving a Fixed Cost

For car sharing:

Total cost = fixed cost + cost depending on distance driven

The co-op pays for gas and other expenses.

Carlo prepared a table to show the total cost to use a car for a day.
Some parts of the table are blank because coffee was spilled on it.

Work with a partner to complete the table.

Car Co-op Costs	
Distance driven (km)	Total cost ($)
0	
10	
20	
30	
40	
50	
60	49.00
70	50.50
80	52.00
90	53.50
100	55.00

- What patterns do you see in the table?
- What is the fixed cost?
  What is the cost per kilometre?
- Graph the data.
- Write a rule to calculate the total cost when you know the distance driven.
- Write an equation. Use C for the total cost in dollars and n for the distance driven in kilometres.

Reflect

Compare your completed table with that of another pair of students.

- What strategies did you use to complete the table?
- How do the patterns in the table help you determine the fixed cost? The cost per kilometre?
- Use your equation to check the cost when a car is driven 90 km.

Connect the Ideas

The cost of a pizza with tomato sauce and cheese is \$9.00.
It costs \$0.75 for each additional topping.
This table shows the cost of a pizza with up to 8 additional toppings.

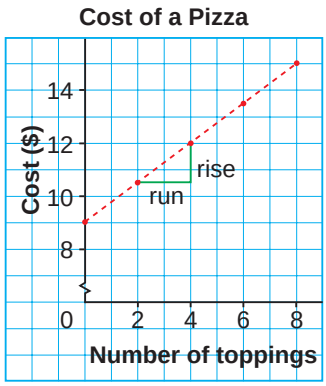
Make a table

Toppings	Cost (\$)
0	9.00
2	10.50
4	12.00
6	13.50
8	15.00

The cost, in dollars, for a 4-topping pizza is $9 + (4 \times 0.75) = 12.00$

Graph the data

The points are joined with a broken line since we cannot order a fraction of toppings.



All points lie on a straight line.
The graph does *not* pass through the origin.

This illustrates **partial variation**.
The cost of a pizza is the sum of a fixed cost and a variable cost.

The point where the line crosses the vertical axis is the **vertical intercept**.
On this graph, the vertical intercept is \$9.00.
It represents the cost of a pizza with 0 extra toppings.

Determine the rate of change

The rise is: \$12.00 − \$10.50 = \$1.50
The run is: 4 toppings − 2 toppings = 2 toppings
The rate of change is: $\frac{\text{rise}}{\text{run}} = \frac{\$1.50}{2 \text{ toppings}}$
$= \$0.75/\text{topping}$

The rate of change is equal to the cost per topping.

Write a rule

Cost of pizza in \$ = 9.00 + (0.75 × number of toppings)

This is the fixed cost in dollars.

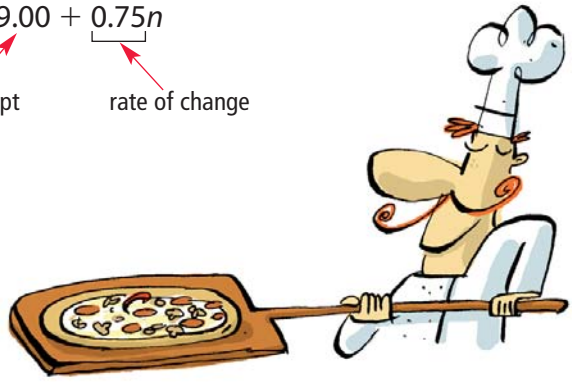
This is the variable cost in dollars because it depends on the number of toppings.

Write an equation

Let C represent the cost in dollars.
Let n represent the number of toppings.
Then, $C = 9.00 + 0.75n$

vertical intercept

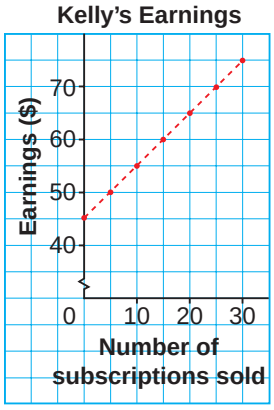
rate of change



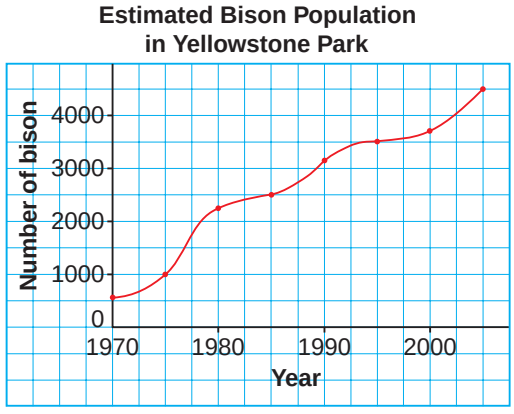
Practice

1. Which graphs represent partial variation? How do you know?

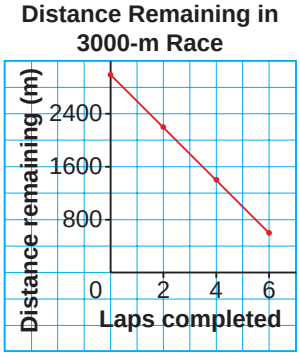
a)



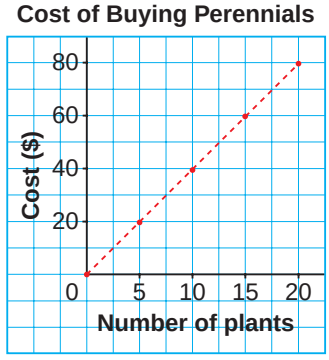
b)



c)



d)



2. For each graph in question 1 that represents partial variation, determine the vertical intercept and the rate of change.

3. When it is windy, it feels colder than the actual temperature. The table shows how cold it feels at different temperatures when the wind speed is 10 km/h.

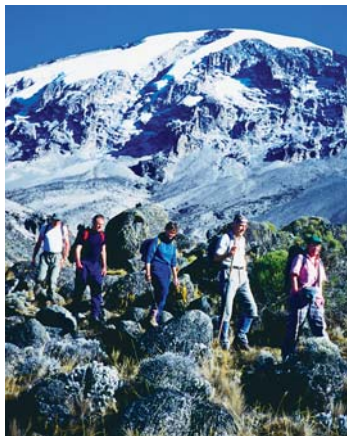
Actual temperature (°C)	−20	−15	−10	−5	0	5
Wind chill equivalent temperature (°C)	−27	−21	−15	−9	−3	3

a) Graph *Wind chill equivalent temperature* against *Actual temperature*.

b) Does this situation represent partial variation? Explain.

c) Use the graph.
About how cold does it feel when there is a 10-km/h wind and the actual temperature is −14°C? 3°C?

4. The temperature at which water boils depends on the altitude; that is, the height above sea level. The table shows how the two quantities are related.



Altitude (m)	0	1200	2400	3600	4800	6000
Boiling point (°C)	100	96	92	88	84	80

a) Graph *Boiling point* against *Altitude*.
Did you join the points with a broken line or a solid line? Explain.

b) Does this situation represent partial variation? Explain.

c) Every year, tourists travel to Tanzania to climb Mount Kilimanjaro. Trekkers must boil their drinking water.
Use the graph to estimate the boiling point of water at each camp:

i) Machame camp, 3100 m

ii) Barafu camp, 4600 m

5. A school is planning an athletic banquet.
The banquet hall charges a fixed cost of $500, plus $25 per person for the food.

a) Make a table to show the total cost for up to 50 guests.

b) Graph *Cost* against *Number of guests*.
Did you join the points with a broken line or a solid line? Explain.

c) Does the graph represent a partial variation? How do you know?

d) What is the rate of change? What does it represent?

e) How much would the banquet cost for 45 guests? 90 guests? How do you know?

f) Is the cost for 45 guests one-half the cost for 90 guests? Explain.

You have determined values by using a graph.
This example shows how to determine values using an equation.

Example

Jolanda has a window cleaning service.
She charges a \$12 fixed cost, plus \$1.50 per window.

a) Write an equation to determine the cost when the number of windows cleaned is known.

b) What does Jolanda charge to clean 11 windows?

Solution

a) The cost is \$12.00 plus \$1.50 $\times$ the number of windows.
Let C represent the cost in dollars.
Let n represent the number of windows.
The equation is: $C = 12.00 + (1.50 \times n)$
or, $C = 12.00 + 1.50n$

b) Use the equation: $C = 12.00 + 1.50n$
To determine the cost for 11 windows, substitute: $n = 11$

$C = 12.00 + 1.50 \times 11$
$= 12.00 + 16.50$
$= 28.50$

Use the order of operations.

Jolanda charges \$28.50 to clean 11 windows.

6. A phone company charges \$20 per month, plus \$0.10 per minute for long distance calling.

a) Write an equation to determine the total monthly charge, C dollars, for n minutes of long distance calling.

b) Determine the total cost for 180 min of long distance calls one month.

7. **Assessment Focus** Nuri is growing his hair to donate to a charity.
The charity makes wigs for children with cancer.
Nuri's hair is now 25 cm long. It grows 1.3 cm every month.

a) Write an equation to determine Nuri's total hair length, h centimetres, after n months.

b) How long is his hair after 6 months? 9.5 months? How do you know?

c) Write a question that can be answered using the equation.
Trade questions with a classmate. Answer the question you receive.

8. Look back at the situations and the equations that represent them.
Use these to help you with this question.

a) Describe a situation that could be modelled by each equation.

i) $C = 150 + 20n$ **ii)** $\ell = 20w$ **iii)** $C = 0.75m$

b) Which equations in part a represent direct variation? Partial variation? How do you know?

9. A school theatre group has posters printed for a performance.
The cost to print a poster is the sum of a fixed cost and a variable cost.
It costs $75 to print 100 posters and $150 to print 400 posters.

a) Graph *Cost* against *Number of posters*.
b) What is the fixed cost? How can you determine this from the graph?
c) What is the rate of change? What does it represent?
d) Write an equation that describes this linear relation.
e) How much does it cost to print 250 posters? How did you find out?

10. A taxi company charges a fixed cost, called the initial meter fare, and a cost per kilometre.
A 5-km trip costs $11.25 and a 10-km trip costs $19.75.

a) Graph *Cost* against *Distance*.
b) What is the fixed cost?
How can you determine this from the graph?
c) What is the rate of change? What does it represent?
d) Write an equation that describes this linear relation.
e) How much does it cost to ride 15 km?
f) This company offers a flat rate fare of $90 for travel from Waterloo to Lester B. Pearson International Airport.
The airport is 96 km from Waterloo.
Which is cheaper: the flat rate fare or the partial variation fare?
How do you know?

11. **Take It Further** In a scavenger hunt, players have 1 h to find 50 items from a list.
Players are awarded 5 points for every item handed in.
Players who hand in their items within 45 min are awarded a 20-point bonus.

a) Suppose a player finishes within 45 min.
  i) Is her score an example of partial variation? Explain.
  ii) Write an equation to determine the total number of points when the number of items is known.
b) Yvonne finishes within 45 min. She scores 220 points.
Determine how many items Kevin needs to find to beat Yvonne in each case.
  i) Kevin also finishes within 45 min.
  ii) Kevin takes the full hour.

In Your Own Words

What is partial variation?
Use an example and a graph to explain.

Using a Graphing Calculator to Graph an Equation and Generate a Table of Values

You will need a TI-83 or TI-84 graphing calculator.

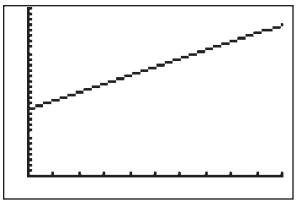
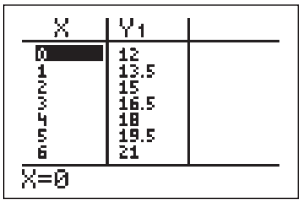
Consider the *Guided Example* in Section 6.5, page 215.
Here is the equation for Jolanda's window cleaning service:
$C = 12.00 + 1.50n$
where C dollars is the cost and n is the number of windows

The graphing calculator uses X to represent the variable on the horizontal axis and Y to represent the variable on the vertical axis.
To match this style, the cost equation is written as:
$Y = 12.00 + 1.50X$

Since X represents the number of windows washed and Y represents the cost, we are only interested in positive values of X and Y.

Follow these steps to graph the equation and generate a table of values.

To make sure there are no equations in the [Y=] list: Press [Y=]. Move the cursor to any equation, then press [CLEAR]. When all equations have been cleared, position the cursor to the right of the equals sign for Y1. To enter the equation, press: [1] [2] [.] [0] [0] [+] [1] [.] [5] [0] [X,T,Θ,n]	Plot1 Plot2 Plot3 \Y1=12.00+1.50X \Y2= \Y3= \Y4= \Y5= \Y6= \Y7=
To adjust the window settings to display the graph: Press [WINDOW]. Set the options as shown. To set an option, clear what is there, enter the value you want, then press [ENTER]. Use the ⊖ key to scroll down the list.	WINDOW Xmin=0 Xmax=10 Xscl=1 Ymin=0 Ymax=30 Yscl=1 Xres=1

Press GRAPH. A graph like the one shown here should be displayed. The points lie on a line. Since X is a whole number, the graph should be a series of dots joined with a broken line. However, the calculator joins the points with a solid line.	
To display a table of values: Press 2nd WINDOW to display the table setup screen. Press 0 ENTER. This sets the initial value of X in the table to 0. Press 1 ENTER. This increases the value of X by 1 each row. Press ENTER ⊖ ENTER. Your screen should look like the one shown here.	
Press 2nd GRAPH to display the table. Only the first 7 rows are displayed. Use the ⊖ key to scroll down the table. Confirm that when 11 windows are washed, the cost is $28.50.	

In Section 6.5, question 8, page 215, you described situations that could be modelled by each of 3 equations.
Complete these steps for each equation.

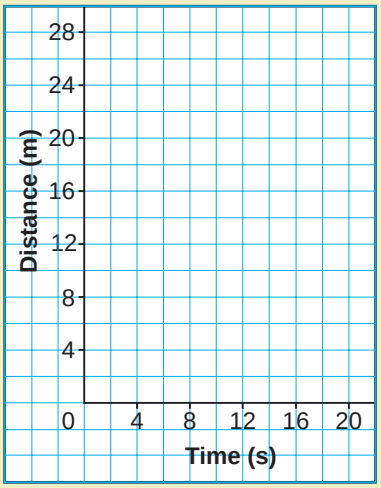
Use pages 217 and 218 as a guide.

- Use a TI-83 or TI-84 calculator to graph the equation, using appropriate window settings.
- Display a table of values for the graph.
- Choose a value of X that makes sense for the situation you described in question 8.
  Use the table to determine the corresponding value of Y.
  Describe what the values represent.

The 25-m Sprint

GAME

In track and field, the 60-m sprint is the shortest race. In this game, you will be racing to 25 m. But you will not be running. Your challenge is to graph a linear relation that reaches 25 m as quickly as possible.



Play with a partner.
You will need 3 number cubes, a ruler, several copies of the grid shown at the right, and a different coloured pencil for each player.
The first player rolls the number cubes and records the numbers shown.
The player chooses:

- one number to be the vertical intercept
- another number to be the rise
- the third number to be the run

Remember that the rise is the vertical distance between two points and the run is the horizontal distance between these points.

The first player graphs her relation:
Plot the vertical intercept.
Use the rise and run to plot the next point.
Plot several points.
Use a ruler to draw a line through them.
Extend the line until it passes beyond 25 m.

Play passes to the second player.
This player rolls the number cubes and decides what the numbers will represent.
The player follows the steps above to graph his relation on the same grid, using a different colour.

Compare the 2 lines.
Whose line reached 25 m first?
This person is the winner.

Play the game several times.
Think about the strategies you use when you decide what the numbers rolled will represent. Describe a winning strategy.
Test your ideas by playing the game again.

Mid-Chapter Review

6.1 6.3

1. The table shows how the height of a small plane changes with time.

Time (min)	Height (m)
0	0
1	200
2	400
3	600
4	800

a) Determine the first differences. What do they represent?
b) Is the relationship linear? Explain.
c) Graph *Height* against *Time*. Determine the rate of change. How does it compare with the first differences?
d) At this rate, how many minutes will it take the plane to reach its cruising height of 1600 m? Explain how you found your answer.

6.2

2. The fastest elevator in the world is in the Taipei 101 office tower, Taiwan. The tower is about 510 m high. It is the tallest building in the world. It takes the elevator about 30 s to reach the top.
a) Graph *Height* against *Time* for the elevator.
b) Determine the height after each time.
i) 15 s ii) 20 s
Which method did you use to find the heights?
c) What is the rate of change of distance with time?
d) What is the average speed for the trip?

6.4 6.5

3. The cost of renting a kayak is $33 per day.
a) Copy and complete this table to show the cost to rent a kayak for up to 10 days.

Rental time (days)	Cost ($)

b) Graph *Cost* against *Rental time*.
c) Does this graph represent direct variation or partial variation? Explain how you know.
d) Determine the rate of change. What does it represent?

6.5

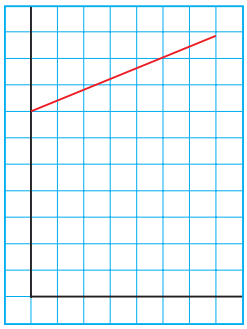
4. In a cookbook, the time to cook a turkey is given as:
30 min per kilogram of turkey, plus an additional 15 min
a) Complete a table like the one below for turkeys with masses up to 10 kg.

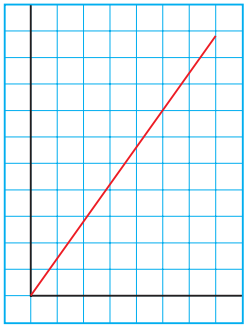
Mass of turkey (kg)	Cooking time (min)
0	15
2	
4	

b) Graph *Cooking time* against *Mass*. Use the graph to determine the time to cook a 5-kg turkey.
c) Write a rule for the cooking time.
d) Write an equation for the cooking time, t minutes, for a turkey with mass m kilograms.
e) Determine the cooking time for a 7.2-kg turkey.
How many different ways could you find this time? Explain.

6.6 Changing Direct and Partial Variation Situations

Investigate Making Changes to Linear Relations





Your teacher will give you a copy of the graphs at the left.

➢ Determine whether each graph represents direct variation, partial variation, or neither. Give reasons for your choices.

➢ Match each graph with its description below.
Label your copy of the graphs with as much detail as possible.
- The "Creature Comforts" dog kennel costs $28 per day.
- Rana rents a moped while on vacation.
  She is charged an initial fee of $35 plus $2 for each litre of fuel consumed.

➢ Write an equation for each graph.

➢ Suppose the daily cost at the dog kennel increases to $30.
How does the graph change?
Sketch the new graph on the grid.
How does the equation change?
Write the new equation.

➢ When Rana rented a moped the next year, the prices had changed.
The initial fee had been reduced to $28.
The price for fuel had increased to $3 per litre.
How does the graph change?
Sketch the new graph on the grid.
How does the equation change?
Write the new equation.

Reflect

Compare your answers with those of other students.

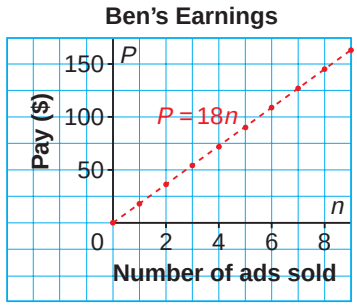
➢ How are graphs of direct variation and partial variation alike? How are they different?

➢ How did the changes in each situation affect the graph?

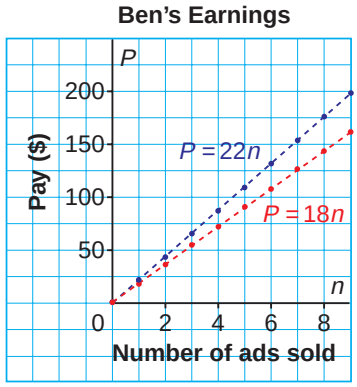
➢ How do these changes affect the equation?

Connect the Ideas

Ben works part time selling newspaper ads.
He is paid \$18 for each ad he sells.
His pay varies directly as the number of ads sold.
The rate of change is \$18/ad.
The rule is:
Pay in dollars = 18 × number of ads sold
Let P represent the pay in dollars.
Let n represent the number of ads sold.
Then the equation is:
$P = 18n$



Ben is given a pay raise.
He is now paid \$22 for each ad he sells.
The rate of change is now \$22/ad.
Here is the new graph and the original graph.
The rule for the new graph is:
Pay in dollars = 22 × number of ads sold
The new equation is:
$P = 22n$



Both graphs pass through the origin and go up to the right.
The purple graph has the greater rate of change.
The purple graph is steeper than the red graph.
Beyond the origin, the purple graph lies above the red graph.

Practice

1. Does each situation represent direct variation or partial variation? Explain how you know.

a) Lily is paid \$5 per hour for raking leaves.

b) The printing of brochures costs \$250, plus \$1.25 per brochure.

c) Jordan is paid \$30 per day, plus \$2.00 for every magazine subscription he sells.

2. Does each equation represent direct variation or partial variation? Explain how you know.

a) $C = 4n + 30$ **b)** $P = 4s$

c) $d = 65t$ **d)** $d = 400 - 85t$

3. The cost for Best Cellular cell phone plan is \$0.20 per minute.

a) Does the cost represent direct variation or partial variation? How do you know?

b) Make a table for the costs. Use intervals of 10 min from 0 min to 100 min.

c) Graph the data.

d) Write an equation to determine the cost, C dollars, for t minutes of calls.

e) The company increases its fees to \$0.25 per minute.

i) How does the graph change? Draw the new graph on the grid in part c.

ii) How does the equation change? Write the new equation.

Changes can be made to a situation described by a partial variation.

Example

Jocelyn works full time selling newspaper ads.
She is paid \$32 a day, plus \$8 for every ad she sells.

a) Make a table of Jocelyn's earnings for daily sales up to 60 ads.

b) Graph the data.

c) Write an equation to determine Jocelyn's earnings, E dollars, when she sells n ads.

d) There is a change in management.
Jocelyn is now paid \$30 per day and \$10 for every ad she sells.

i) Graph Jocelyn's earnings for daily sales up to 60 ads.
How is the graph different from the graph in part b?

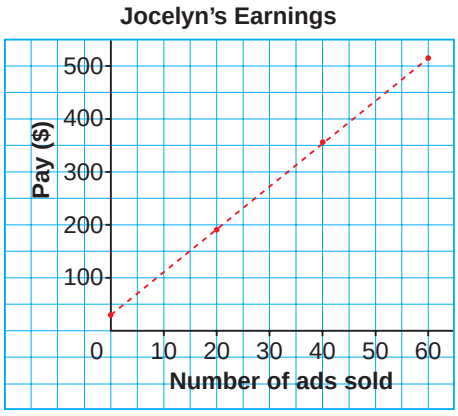
ii) Write the new equation for Jocelyn's earnings.

Solution

a)

Number of ads	Earnings (\$)
0	32
20	$32 + (8 \times 20) = 192$
40	$32 + (8 \times 40) = 352$
60	$32 + (8 \times 60) = 512$

b)

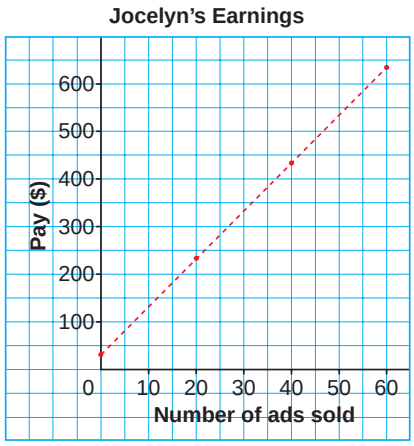


The vertical intercept is \$32.
The rate of change is \$8/ad.

c) The rule for Jocelyn's earnings is:
Earnings in dollars $= 32 + (8 \times$ number of ads sold$)$
An equation is: $E = 32 + 8n$

d) i) Here is the graph with the new rate. The vertical intercept is \$30. The rate of change is \$10/ad.

ii) The new equation is: $E = 30 + 10n$



4. Use the data in the *Guided Example*. Which pay scale was better for Jocelyn? Give reasons for your answer.

5. The Best Cellular cell phone company has another plan for cell phone users. This plan costs \$12 per month, plus \$0.10 per minute.

a) Make a table of monthly costs for calling times from 0 min to 100 min.

b) Graph the data.

c) Write an equation to determine the monthly cost, C dollars, for t minutes of calls.

d) The company increases its fixed monthly fee to \$20 but decreases its cost per minute to \$0.08.

i) How does the graph change? Draw the new graph on the grid in part b.

ii) Write the new equation.

6. **Assessment Focus** Use the data and your results from questions 3 and 5. For each plan, does the cost double when the time doubles? Explain.

7. **Take It Further** A car rental agency charges a daily rate of \$29. If the distance driven is more than 100 km, a charge of \$0.10 per kilometre is added for each kilometre over 100 km.

a) Make a table to show the cost every 50 km up to 300 km.

b) Graph the data.

c) Does the graph represent partial variation, direct variation, or neither? How do you know?

d) Describe the graph. Explain its shape.

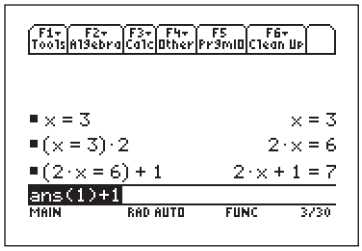
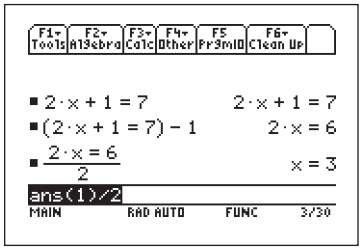
e) How much would you have to pay if you drove 35 km? 175 km? 280 km?

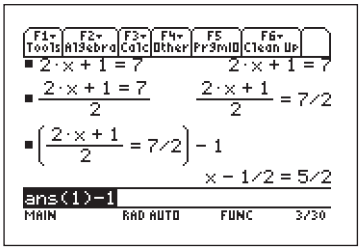
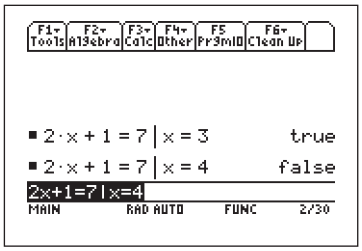
In Your Own Words

How are the graph and equation of a partial variation situation different from those of direct variation? Use examples in your explanation.

Using CAS to Explore Solving Equations

You will need a TI-89 calculator.

Step 1 ➢ Clear the home screen: HOME F1 8 CLEAR ➢ Enter the equation $x = 3$: X = 3 ENTER ➢ Multiply each side of the equation by 2: × 2 ENTER ➢ Add 1 to each side of the equation: + 1 ENTER	At the end of Step 1, your screen should look like this:  	$x = 3$ $2x = 6$ $2x + 1 = 7$ are examples of *equivalent equations*. Why are the equations equivalent? How are equivalent equations formed?
Step 2 In Step 1, you started with $x = 3$ and ended with $2x + 1 = 7$. Suppose you start with $2x + 1 = 7$. What steps would you take to get back to $x = 3$?		Which operations did you use to get from $x = 3$ to $2x + 1 = 7$? How could you "undo" these operations to get back to $x = 3$?
Step 3 ➢ Clear the home screen: HOME F1 8 CLEAR ➢ Enter the equation $2x + 1 = 7$: 2 X + 1 = 7 ENTER ➢ Subtract 1 from each side: − 1 ENTER ➢ Divide each side by 2: ÷ 2 ENTER	At the end of Step 3, your screen should look like this:  	The steps to get back to $x = 3$ are the steps to solve the equation $2x + 1 = 7$. At each step: – we are closer to isolating the term containing the variable – we create a simpler equivalent equation

Step 4 ➢ Clear the home screen: [HOME] [F1] 8 [CLEAR] ➢ Enter $2x + 1 = 7$: 2[X] [+] 1 [=] 7 [ENTER] ➢ Divide each side by 2: [÷] 2 [ENTER] ➢ Subtract 1 from each side: [−] 1 [ENTER]	At the end of Step 4, your screen should look like this:  	Which extra step is needed to isolate x on the left side of the equation? What is the result when you isolate x? Which method, Step 3 or Step 4, is easier to use? Explain.
Step 5 ➢ Work with a partner. Repeat Step 1 with a different start equation. Multiply each side by a different number. Then add a different number to each side, or subtract a different number from each side. ➢ Trade end equations with your partner. ➢ Solve your partner's equation.		Which strategies did you use to solve the equations?
Step 6 Solve these equations using paper and pencil. **a)** $3x - 1 = 14$ **b)** $5x + 12 = 2$ **c)** $15 - 2x = 3$ Use CAS to check your work and to check your solution.	To check if $x = 3$ is a solution to $2x + 1 = 7$, press: 2[X] [+] 1 [=] 7 [|] [X] [=] 3 [ENTER]. To test $x = 4$, press: [▶] [←] 4 [ENTER]  	How does the calculator show that $x = 3$ is a solution? How does it show that $x = 4$ is *not* a solution?

6.7 Solving Equations

An equation is a statement that two expressions are equal.
Each of these is an equation:

$3x = 21$

$12 = x + 4$

$2x - 7 = 9$

Investigate Using Inverse Operations to Solve an Equation

Work with a partner.

- Start with the equation $x = 2$.
  This is the *start equation*.
- Multiply each side of the equation by 3.
  Write the resulting equation.
- Add 5 to each side of the equation.
  Write the resulting equation.
  This is the *end equation*.
- Suppose you were given the end equation.
  What steps would you take to get back to $x = 2$?

Repeat the steps above with a different start equation.
Multiply each side by a different number.
Then add a different number to each side,
or subtract a different number from each side.
Trade end equations with your partner.
Find your partner's start equation.

Reflect

Share your equations with another pair of classmates.

- Find your classmates' start equations.
  What strategies did you use?
- How are the operations used to get from the end equation to the start equation related to the operations used to get from the start equation to the end equation?

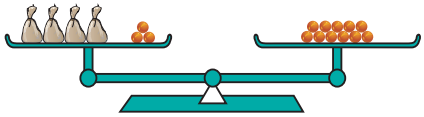
Connect the Ideas

We can use mental math to solve equations like $3 + x = 5$.
We know that $3 + 2 = 5$, so $x = 2$.

Equations like $4x + 3 = 11$ cannot be easily solved mentally.
Here are two ways to solve the equation.

Use a balance scales model

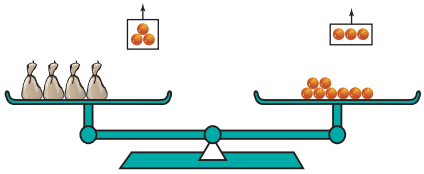
Let x represent the number of candies in each bag.
There are 4 bags and 3 candies on the left pan.



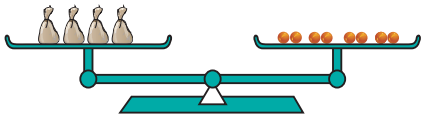
This is represented by the expression $4x + 3$.
There are 11 candies on the right pan.
This is represented by the number 11.
So, the balance scales model the equation $4x + 3 = 11$.

Each bag contains the same number of candies.

To solve the equation:
Remove 3 candies from each pan.
We now have: $4x = 8$



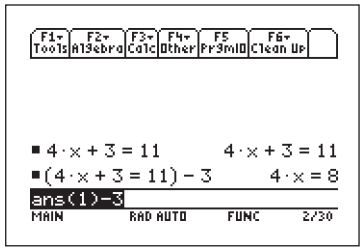
Divide the candies in the right pan into 4 equal groups.
Each group contains 2 candies.
So, each bag contains 2 candies.
The solution is $x = 2$.



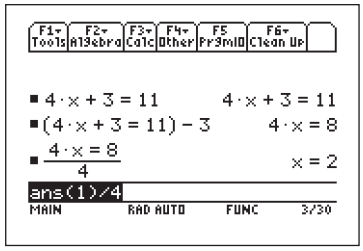
Use computer algebra systems (CAS)

Enter the equation $4x + 3 = 11$.
To solve the equation, use inverse operations.

Isolate the term $4x$ by "undoing" the addition.
The inverse of adding 3 is subtracting 3.
Subtract 3 from each side of the equation.
We now have: $4x = 8$



Isolate x by "undoing" the multiplication.
The inverse of multiplying by 4 is dividing by 4.
Divide each side of the equation by 4.
The solution is $x = 2$.



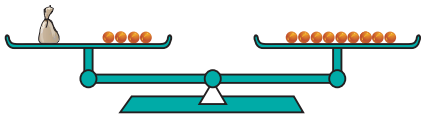
Practice

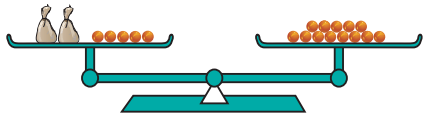
1. Solve each equation. Explain how you did it.

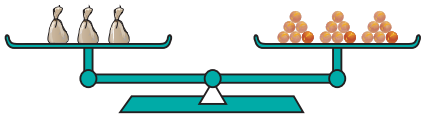
a) $3x = 12$ b) $45 = 5x$ c) $x - 7 = 11$

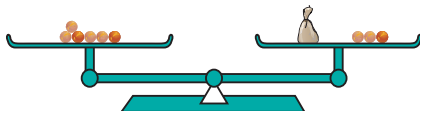
d) $2x = 4$ e) $4 = 3 + x$ f) $x + 5 = 14$

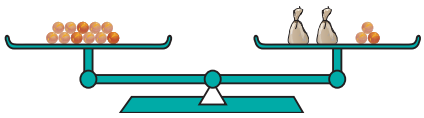
2. Write the equation represented by each balance scales. Explain how to solve the equation.

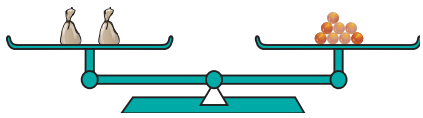
a)


b)


c)


d)


e)


f)


3. Solve each equation.

a) $3x + 2 = 17$ b) $2x - 5 = 1$

c) $33 = 4x + 5$ d) $7x + 1 = 22$

e) $2 = 6x - 10$ f) $5x - 2 = 38$

Which tools could you use to help you?

Having trouble?

Read Connect the Ideas.

We cannot use balance scales when the solution to an equation is a negative number. Use inverse operations instead.

Example

Solve: $3x + 14 = 2$

Solution Use an inverse operation to isolate $3x$ on the left side of the equation.

$3x + 14 = 2$ Subtract 14 from each side.

$3x + 14 - 14 = 2 - 14$

$3x = -12$ To isolate x, divide each side by 3.

$\frac{3x}{3} = \frac{-12}{3}$

$x = -4$

Check the solution.

Substitute $x = -4$ in $3x + 14 = 2$.

L.S. $= 3(-4) + 14$ R.S. $= 2$

$= -12 + 14$

$= 2$

Since the left side equals the right side, $x = -4$ is the correct solution.

4. Solve each equation.

a) $3x + 4 = 1$ **b)** $5 + 2x = 3$ **c)** $-3x + 4 = 7$ **d)** $2 - 5x = 17$

e) $7 = 3x + 13$ **f)** $-3x - 5 = 4$ **g)** $24 = -3x - 6$ **h)** $-5 + 4x = -21$

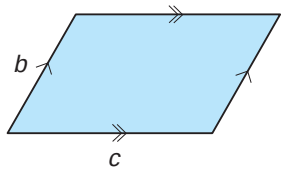
5. Choose 3 equations in question 4. Check your solutions.

6. An equation for the perimeter of a parallelogram is $P = 2b + 2c$.

a) Determine P when $b = 5$ cm and $c = 7$ cm.

b) Determine c when $P = 36$ cm and $b = 8$ cm

c) Determine b when $P = 54$ cm and $c = 12$ cm.



7. **Assessment Focus** In Canada, temperature is measured in degrees Celsius, C.
In the United States, temperature is measured in degrees Fahrenheit, F.
The equation $9C = 5F - 160$ is used to convert between the two temperature scales.

a) What is 30°C in degrees Fahrenheit?

b) What is 14°F in degrees Celsius?

Explain your work.

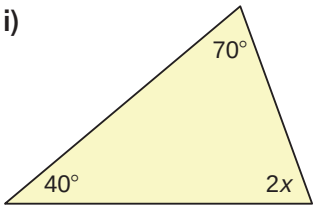
8. An empty tanker truck has a mass of 14 000 kg.
One barrel of oil has a mass of 180 kg.
The equation $M = 14\,000 + 180b$ represents the total mass of the truck, M kilograms, when it contains b barrels of oil.
The truck enters a weigh station that shows its total mass is 51 080 kg.
How many barrels of oil are on the truck?
How do you know?

9. **Take It Further** In each figure:

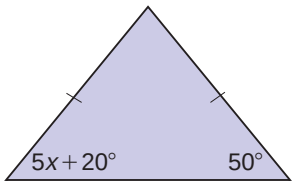
a) Write an equation to relate the given measures.

b) Solve the equation.

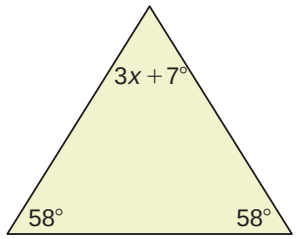
i)



ii)



iii)



In Your Own Words

Choose one equation you solved in this section.
Explain how you solved it.
Show how to check the solution.

6.8 Determining Values in a Linear Relation

In forestry, the diameter of a tree is measured at chest height, 1.37 m above the ground.

To estimate how much cut lumber a tree can provide, a forester needs to know the size of the tree.
It can be difficult to measure the height of a tree.
It is easier to measure the tree's circumference, then calculate its diameter.
So, it is useful to know how a tree's height is related to its diameter.

Investigate Determining Values Using a Table, Graph, and Equation

The relationship between height and diameter changes as trees age. This equation is valid for trees with diameters between 5 cm and 25 cm.

Researchers collected data from more than 100 white spruce trees on Prince Edward Island.
The data were graphed and a line of best fit was drawn.
The equation of the line is: $d = 10h - 50$
where d is the diameter of the tree in centimetres and h is the approximate height of the tree in metres

- Make a table to show the approximate heights of trees with diameters 5 cm and 25 cm.

Approximate height, h (m)	Diameter, d (cm)
	5
	25

- Graph the data.
  How could you use the graph to estimate the height of a tree that is not in the table?
- How could you use the graph to estimate the diameter of a tree that is not in the table?

Reflect

You have used a table, a graph, and an equation to estimate the heights and diameters of trees.
Which method do you prefer? Is one method best? Explain.

Connect the Ideas

Janice is a piano teacher.
She earns $23 per hour.
Janice wants to earn $200.
Here are three ways she can determine the time she has to work.

Make a table

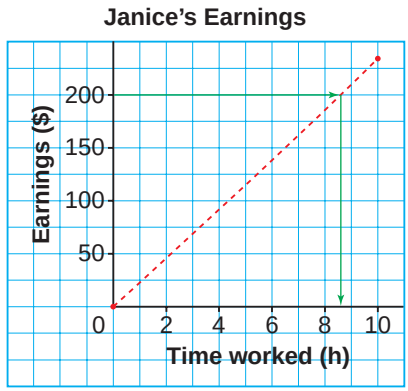
Continue the table until the *Earnings* is $200 or greater.

Time worked (h)	1	2	3	4	5	6	7	8	9
Earnings ($)	23	46	69	92	115	138	161	184	207

From the table, Janice has to work 9 h to earn at least $200.

Draw a graph

Janice earns $23/h.
She earns $0 for 0 h worked.
Janice earns $23 × 10 = $230 for 10 h worked.
Since the relationship is a direct variation, plot the points (0, 0) and (10, 230), then draw a broken line through them.
From the graph, Janice has to work about 8.5 h to earn $200.



Use an equation

Since the relationship is a direct variation, the rule is:
Earnings in dollars = rate of change × time worked in hours
The rate of change is the hourly rate: $23/h
Let A dollars represent the earnings.
Let n hours represent the time worked.
Then, the equation is: $A = 23n$
To determine how long Janice needs to work to earn $200, substitute $A = 200$ and solve the equation for n.
Use inverse operations.

$200 = 23n$ Divide each side by 23.

$\frac{200}{23} = \frac{23n}{23}$ Use a calculator to determine 200 ÷ 23.

$8.696 \doteq n$

Janice has to work about 8.7 h to earn $200.
Since Janice works whole numbers of hours, she has to work 9 h to earn $200.

Practice

1. **a)** This table shows the cost to rent a canoe.
How much would it cost to rent a canoe for each time?
i) 2 days **ii)** 4 days
How did you find out?

Time (days)	0	1	3	5
Cost ($)	0	24	72	120

b) This table shows the earnings from ticket sales.
How much is earned from the sale of each number of tickets?
i) 10 **ii)** 30 **iii)** 50
How did you find out?

Tickets sold	0	20	40	60
Earnings ($)	0	160	320	480

2. These data were recorded for a cheetah chasing prey.

Time (s)	0	5	10	15	20
Distance (m)	0	105	210	315	420

a) Graph the data.
b) Use the graph to determine how long it takes the cheetah to run 400 m.
c) About how long will it take the cheetah to run 500 m? What assumption did you make?
d) Write an equation to determine the distance, d metres, after t seconds.
e) Use the equation to determine how long it will take the cheetah to run 400 m and 500 m. How do your answers compare to those in parts b and c? Which method did you prefer? Explain.

We can also find values for a linear relation that represents partial variation.

Example

A candle is 13 cm long.
It is lit and it burns at the rate of 0.2 cm/min.
a) Write an equation for the height, H centimetres, of the candle after time, t minutes.
b) How long does it take until the candle is 10 cm high?

Solution

a) The rule is:
Candle height = initial height − (rate of change × time)
The range of change is 0.2 cm/min.
So, the equation is:
$H = 13 - (0.2 \times t)$, or
$H = 13 - 0.2t$

b) To determine how long it takes until the candle is 10 cm high, substitute $H = 10$ in the equation, then solve for t.

$$H = 13 - 0.2t \quad \text{Use inverse operations.}$$
$$10 = 13 - 0.2t \quad \text{Subtract 13 from each side.}$$
$$10 - 13 = 13 - 13 - 0.2t$$
$$-3 = -0.2t \quad \text{Divide each side by } -0.2.$$
$$\frac{-3}{-0.2} = \frac{-0.2t}{-0.2} \quad \text{Use a calculator to divide.}$$
$$15 = t$$

It takes 15 min until the candle is 10 cm high.

3. Use the data in the *Guided Example*.

a) Graph *Height* against *Time*.
Determine when the candle is 10 cm high. How did you do this?

b) Make a table of values.
Use the table to determine when the candle is 10 cm high.

c) Which of the 3 methods to determine the height of the candle did you prefer? Explain.

4. **Assessment Focus** At the video arcade, Aran bought a game card with a value of $35. Every time he plays a game, $1.40 is deducted from the card.

a) Make a table for the number of games played.
Use the table to determine when the card runs out.

Number of games played, n	Value of card, V ($)

b) Graph the data.
Use the graph to determine when the card runs out.

c) Write an equation for the value, V dollars, of the card after n games have been played.

d) How many games can Aran play before his card runs out? How do you know?

e) Which of the 3 methods do you prefer? Explain.

5. **Take It Further** Aran could have bought a card that costs $10 more, but only $0.90 is deducted for each game played.

a) How does this affect the graph?

b) How many more games could Aran play with this card?

c) Do you think it is a good deal? Explain.

In Your Own Words

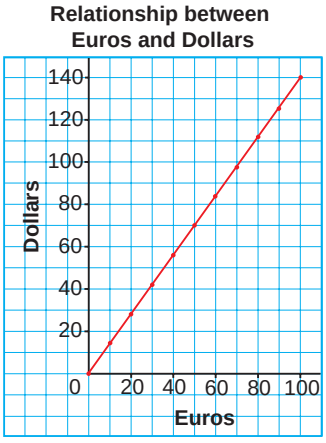
Describe some different ways to determine values in a linear relation. Which way do you prefer? Use an example to explain.

6.9 Solving Problems Involving Linear Relations

Vimy Ridge, in Northern France, was the site of one of the most infamous battles of World War I.
In 2007, groups of Canadian high school students will visit the site to commemorate the 90th anniversary of the battle.
When they travel to France, students will convert their money from dollars to Euros.

Investigate Developing an Equation from a Graph

The price of one currency in terms of another is called the *exchange rate*.
Exchange rates vary from day to day.
This graph is based on the average exchange rate in January 2006.
Your teacher will give you a copy of the graph.



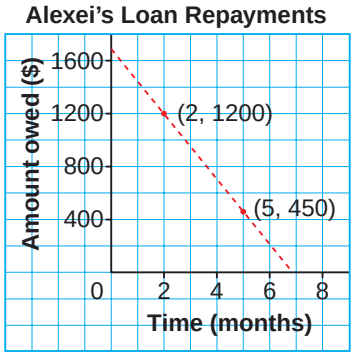
- Does this graph represent direct variation or partial variation? Explain how you know.
- Use the graph.
  Determine the value of $70, in Euros.
  What is the value of 100 Euros, in dollars?
- Determine the rate of change.
  What does it represent?
- Write an equation for the value, *d* Canadian dollars, of *E* Euros.
- Use the equation to determine the value of $100, in Euros.
  What is the value of 20 Euros, in dollars?
  Can you use the graph to determine these values? Explain.

Reflect

- What strategies did you use to write the equation of a relationship given its graph?
- When might you want to use the equation rather than the graph?

Connect the Ideas

Alexei went on a ski trip that cost $1700.
He borrowed the money from his parents to pay for the trip.
Every month, he pays them back the same amount of money.



Determine the rate of change

From the graph
After 2 months, Alexei owes $1200.
After 5 months, he owes $450.
So, the rise is: $450 − $1200 = −$750
The run is: 5 months − 2 months = 3 months

$$\text{Rate of change} = \frac{\text{rise}}{\text{run}}$$
$$= \frac{-\$750}{3 \text{ months}}$$
$$= -\$250/\text{month}$$

This means that every month, Alexei owes $250 less than the month before.
That is, his payments are $250 each month.

Write a rule

The vertical intercept is $1700.
This represents the amount that Alexei borrowed;
that is, the amount when measurements of time began.

Amount owed = $1700 − ($250/month × time in months)

Write an equation

Let A dollars represent the amount.
Let t months represent the time.
Then, an equation is: $A = 1700 - 250t$

We can find out when Alexei will have paid off his loan.

➢ Use the graph.
At 7 months, the amount owed is approximately zero.
This means that Alexei has paid off the loan.

➢ Use the equation.
The amount Alexei owes is \$0.
Substitute $A = 0$, then solve for t.

$$A = 1700 - 250t$$
$$0 = 1700 - 250t \qquad \text{Add } 250t \text{ to each side.}$$
$$250t = 1700 - 250t + 250t$$
$$250t = 1700 \qquad \text{Divide each side by 250.}$$
$$\frac{250t}{250} = \frac{1700}{250}$$
$$t = 6.8$$

When we add $250t$ to each side, then divide each side by 250, we are using inverse operations.

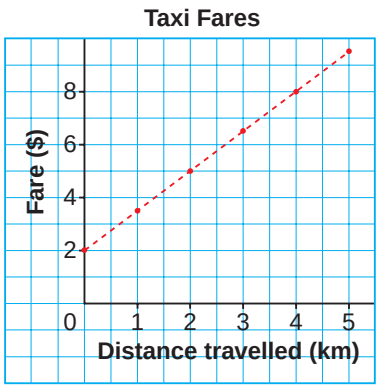
Since the number of months is a whole number, it takes Alexei 7 months to pay off his loan.

Practice

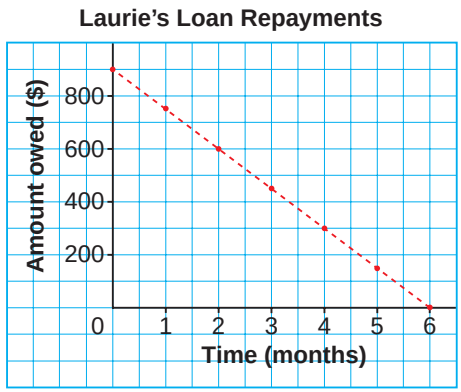
Your teacher will provide larger copies of the graphs in this section.

1. For each graph, make a table of values.

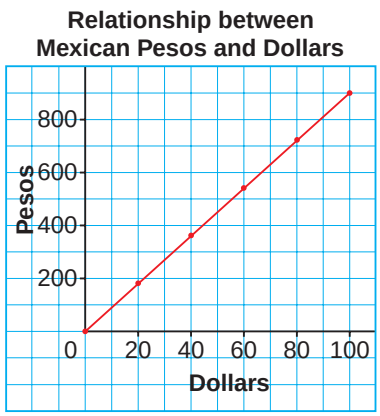
a)



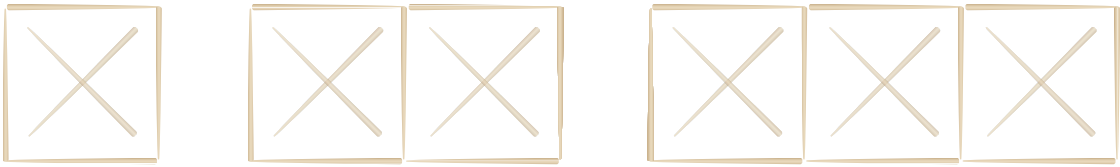
b)



c)

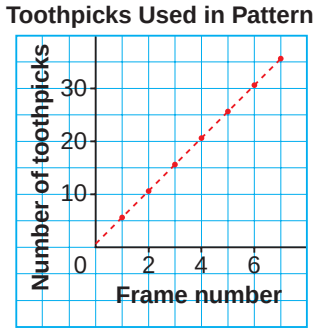


2. Rosa made this pattern with toothpicks. She continued it for 7 frames.



Frame 1 Frame 2 Frame 3

Rosa drew this graph.



a) Does this graph represent direct variation or partial variation?
How do you know?

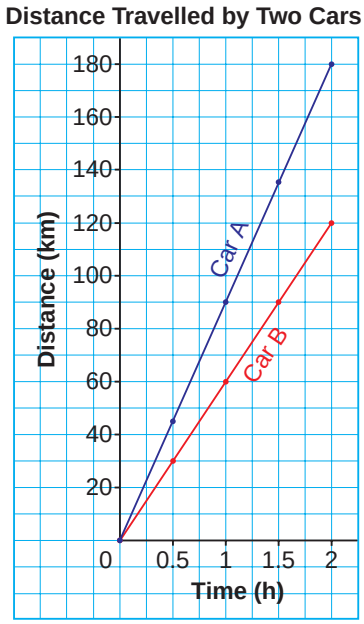
b) Make a table of values to show how many toothpicks are needed for each frame.

c) Write an equation that relates the number of toothpicks, T, and the frame number, n.

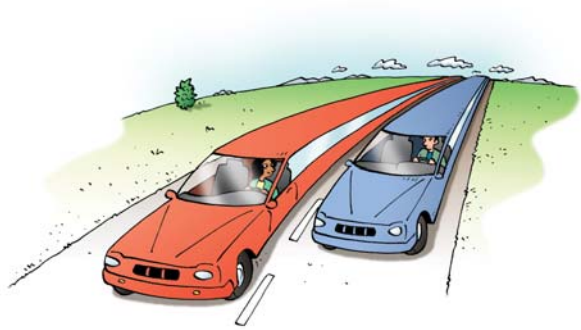
d) How many toothpicks would you need to build the 17th frame in this pattern?
Which method did you use to find out?

3. The graph below models the motion of two cars.

a) Use the graph.
Copy and complete the table.



Time (h)	Car A Distance (km)	Car B Distance (km)
0		
0.5		
1		
1.5		
2		



b) For each car, write an equation that relates distance, d kilometres, and time, t hours.

c) Which car has the greater average speed? How do you know?

d) How far has each car travelled after 1.75 h?

e) How long does it take each car to travel 110 km?

We can graph a relation from its description.

Example

Ashok and Katie each recorded a distance for a ball rolling over a period of time.
Ashok found that the ball rolled 9 m in 3 s.
Katie found that the ball rolled 6 m in 2 s.

a) Draw a distance-time graph for the ball.
Assume the ball continues at the same average speed.

b) When will the ball have rolled 10 m?

c) Write an equation that relates the distance, d metres, and the time, t seconds.

d) Use the equation.
How far did the ball roll in 8 s?

e) Compare your solutions in parts b and d.

Solution

a) Complete a table of values.
Graph the data.

Time, t (s)	0	2	3
Distance, d (m)	0	6	9

The points lie along a straight line that passes through the origin.
Extend the line.

b) To determine when the ball rolled 10 m, use the graph.
From the graph, the ball reached 10 m after about 3.3 s.

c) The situation is modelled by direct variation.
Distance in metres = average speed in m/s × time in seconds
The average speed is the rate of change.
The ball rolled 9 m in 3 s.

$$\text{Average speed} = \frac{\text{distance}}{\text{time}} = \frac{9\text{ m}}{3\text{ s}} = 3\text{ m/s}$$

So, distance in metres = 3 m/s × time in seconds
The equation is: $d = 3t$

The graph will not go on forever. The ball will eventually slow down and stop.

d) To determine how far the ball rolled in 8 s, substitute $t = 8$.

$$d = 3 \times 8 = 24$$

e) In part b, from the graph, the answer was approximate.
To determine the answer, the graph was extended to the right.
Drawing the graph helps visualize the relationship between the quantities.
In part d, from the equation, the answer was exact.

4. Liam sells jewellery.
He earns a basic wage, plus commission.
Liam earned $550 when he sold $1000 worth of jewellery.
He earned $750 when he sold $5000 worth of jewellery.
How much does Liam earn when he sells $6000 worth of jewellery?
a) Use a table to solve the problem.
b) Use an equation to solve the problem.
c) Which method in parts a and b do you prefer? Explain.
d) How could you have solved the problem a different way? Explain.

5. **Assessment Focus** Rain water is collected in a 350-L barrel.
During one storm, the volume of water in the barrel after 1 min was 65 L.
The volume after 5 min was 125 L.
Suppose water continues to be collected at this rate.
a) How many minutes will it take for the barrel to fill?
b) How many different ways could you determine the answer to part a?
Describe each way.
Which way do you prefer?

6. Suppose the rate of water collection in question 5 was 12 L per minute.
How many minutes would it take for the barrel to fill?

7. **Take It Further** Use the table.
a) Write an equation to show the relationship between the distance, *d*, and time, *t*.
b) Describe a situation that could be modelled by this table and equation.

Time, *t* (min)	0	1	2	3	4
Distance, *d* (m)	3	5	7	9	11

In Your Own Words

Describe how to write an equation of a relationship when you are given its graph. Use an example in your answer.

6.10 Two Linear Relations

Speed skating is an Olympic sport that has grown in popularity.

Investigate Exploring a Pair of Relations

Mei and Zachary are junior speed skaters.
Mei skates at an average speed of 10 m/s.
Zachary skates at an average speed of 9 m/s.
Mei and Zachary have a 1500-m race.
Zachary has a head start of 100 m.
Who wins the race? How do you know?

Reflect

- How did you solve the problem?
  If you did *not* draw a graph, work with a partner to use a graph to solve the problem.
- Suppose the race was only 500 m.
  Who would have won? How do you know?
- Suppose the race was 1000 m.
  Who would have won? How do you know?

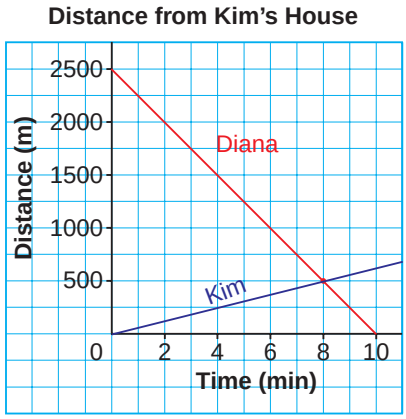
Connect the Ideas

Recall that
1 km = 1000 m
So, 2.5 km = 2500 m

Diana and Kim live 2.5 km apart.
They leave their homes at the same time.
They travel toward each other's house.
Diana is on rollerblades.
She skates at an average speed of 250 m/min.
Kim is walking at an average speed of 60 m/min.
We can draw a graph to determine where and when they will meet.

The table below shows the distance each person is from Kim's house.
Each minute, Kim walks 60 m and Diana skates 250 m.
Graph the data for each person on the same grid.

Time (min)	Kim's distance (m)	Diana's distance (m)
0	0	2500
1	60	2250
2	120	2000
3	180	1750
4	240	1500



How else could this problem have been solved?

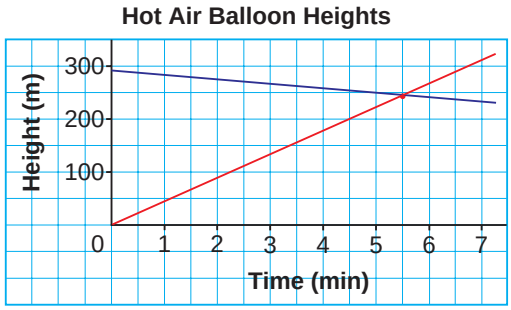
The lines intersect at approximately (8, 500).
The point of intersection shows where and when the people meet: after about 8 min and about 500 m from Kim's house.

Practice

1. One hot air balloon is descending.
Another is ascending.
The graph shows their heights over time.

a) What is the starting height of the descending balloon?

b) Where do the lines intersect? What does this point represent?



2. Nyla and Richard are travelling toward each other along the same bike path.
Nyla jogs at an average speed of 100 m/min.
Richard bikes at an average speed of 300 m/min.
The table shows their distances, in metres, relative to Richard's starting position.

Time (min)	Nyla's distance (m)	Richard's distance (m)
0	4000	0
5		
10		

a) To begin, how far apart are they?
b) Copy and complete the table for times up to 30 min. Graph the data.
c) Where do the lines intersect? What does this point represent?

3. **Assessment Focus** Hatef and Sophie are participating in a swim-a-thon.
Swimmers may complete a maximum of 50 laps.
Hatef has collected pledges totalling \$13 per lap.
Sophie has collected \$96 in cash and pledges totalling \$5 per lap.
a) Make a table to show how much money each swimmer collects for completing up to 50 laps. Graph both sets of data on the same grid.
b) What is the most money each person can collect?
c) Where do the lines intersect? What does this point represent?

Two lines on a graph can be used to help make business decisions.

Example

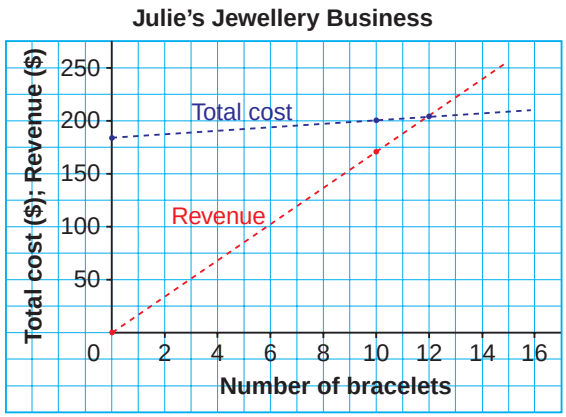
Julie started a business making bracelets.
Her costs are \$180 for tools and advertising, plus \$2 in materials per bracelet. Each bracelet is sold for \$17.
a) Write an equation for the total cost, C dollars, to make n bracelets.
b) Write an equation for the revenue, R dollars, from the sale of n bracelets.
c) Graph both equations on the same grid.
d) Where do the lines intersect? What does this point represent?

Revenue is the money Julie gets when she sells the bracelets.

Solution

a) Total cost = (initial costs) + (materials cost)
The initial costs are \$180.
The materials cost is \$2 per bracelet.
The equation is: $C = 180 + 2n$
b) Revenue = (selling price per bracelet) × (number of bracelets sold)
The selling price is \$17, so $R = 17n$
c) To draw each graph, use the vertical intercept and one other point.
For *Total cost*, the vertical intercept is 180.
Plot the point (0, 180).
Substitute $n = 10$ in the equation $C = 180 + 2n$.
$C = 180 + (2 \times 10) = 200$
Plot the point (10, 200).

Join the points with a broken line and extend the line to the right.
For *Revenue*, the vertical intercept is 0.
Plot the point (0, 0).
Substitute $n = 10$ in the equation $R = 17n$.
$R = 17 \times 10 = 170$
Plot the point (10, 170).
Join the points with a broken line and extend the line to the right.



d) The lines intersect at approximately (12, 200).
This is the point where the total cost equals the revenue.
To the left of this point, the *Revenue* graph is below the *Total cost* graph, so Julie makes a loss.
To the right of this point, the *Revenue* graph is above the *Total cost* graph, so Julie makes a profit.

4. The weekly cost of running an ice-cream cart is a fixed cost of $150, plus $0.25 per ice-cream treat. The revenue is $2.25 per treat.

a) Write an equation for the weekly cost, C dollars, when n treats are sold.

b) Write an equation for the revenue, R dollars, when n treats are sold.

c) Graph the equations on the same grid.

d) How many treats have to be sold before a profit is made? Explain.

5. **Take It Further** Mark reads at an average rate of 30 pages per hour.
Vanessa reads at an average rate of 40 pages per hour.
Mark started reading a book.
Vanessa started reading the same book when Mark was on page 25.

a) How many hours later will they be reading the same page? How do you know?

b) How many different ways can you answer part a? Describe each way.

In Your Own Words

Describe a situation that can be modelled by two lines on a graph.
Explain what the point of intersection represents.

Using a Graphing Calculator to Determine Where Two Lines Meet

Technology

You will need a TI-83 or TI-84 graphing calculator.

Nicole joins a fitness club.
The club offers two types of memberships.
Nicole can pay a monthly fee of \$24, plus \$2 for each visit.
Or, Nicole can pay \$8 per visit.
Which type of membership should Nicole buy?

Let C dollars represent the total monthly cost.
Let n represent the number of times Nicole uses the gym in a month.
The equations are:

$C = 24 + 2n$ $\qquad$ $C = 8n$

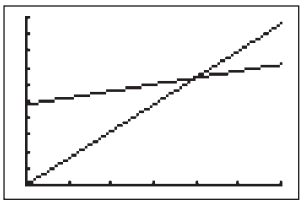
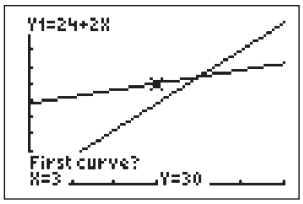
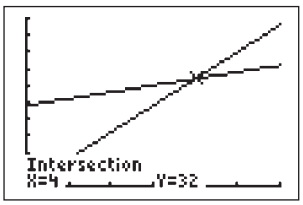
The graphing calculator uses X to represent n and Y to represent C.
To match this style, the equations are written as:

$Y = 24 + 2X$ $\qquad$ $Y = 8X$

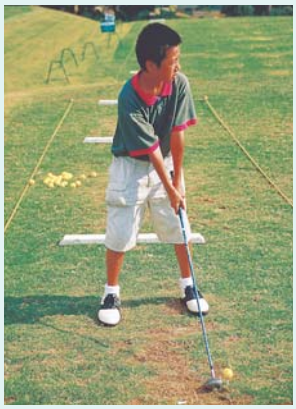
Follow these steps to graph the equations and determine the point of intersection.

To make sure there are no equations in the [Y=] list: Press [Y=]. Move the cursor to any equation, then press [CLEAR]. When all equations have been cleared, position the cursor to the right of the equals sign for Y1. To enter the equations, press: [2] [4] [+] [2] [X,T,Θ,n] [ENTER] [8] [X,T,Θ,n]	Plot1 Plot2 Plot3 \Y1=24+2X \Y2=8X \Y3= \Y4= \Y5= \Y6= \Y7=
To adjust the window settings to display the graph: Press [WINDOW]. Set the plot options as shown. To set an option, clear what is there, enter the value you want, then press [ENTER]. Use the ⊖ key to scroll down the list.	WINDOW Xmin=0 Xmax=6 Xscl=1 Ymin=0 Ymax=50 Yscl=5 Xres=1

We are only interested in positive values of X and Y.

Press GRAPH. A graph like the one shown here should be displayed. The points lie on a line. Since X is a whole number, each line should be a series of dots joined with a broken line. However, the calculator joins the points with solid lines.	
To determine the point of intersection: Press 2nd TRACE to access the CALCULATE menu. Then press 5. Your screen should look like the one shown here. Press ▶ to move the flashing point until it is close to the point of intersection. Press ENTER ENTER ENTER.	
Your screen should look like the one shown here. The coordinates of the point of intersection appear at the bottom of the screen. • Where do the lines intersect? • What does this point represent? • Which type of membership would you recommend to Nicole? Explain your thinking.	

Use pages 245 and 246 as a guide.



Tyresse practises at the driving range to improve his golf swing. The driving range offers two payment options:

- Tyresse can pay $6.50 for each bucket of balls he hits.
- Tyresse can purchase a membership for $24.50. He will then pay $4.75 for each bucket of balls he hits.

➢ Write an equation to represent each payment option.

➢ Use a TI-83 or TI-84 calculator to graph the equations, using appropriate window settings.

➢ Determine where the lines intersect. Explain what the point represents.

➢ Which payment option would you recommend to Tyresse? Explain your thinking.

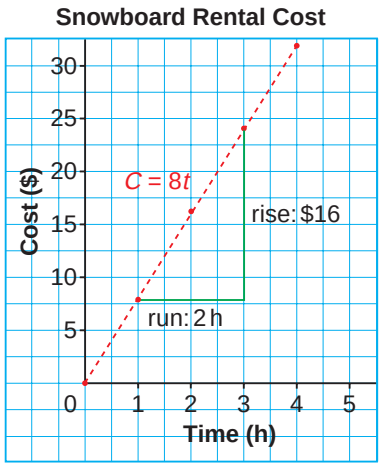
Chapter Review

What Do I Need to Know?

Both direct variation and partial variation are linear relations.

➢ Direct Variation

A graph that represents direct variation is a straight line that passes through the origin.



Time t (h)	Cost C ($)	First Differences
0	0	
1	8	$8 - 0 = 8$
2	16	$16 - 8 = 8$
3	24	$24 - 16 = 8$
4	32	$32 - 24 = 8$

Rate of change $= \frac{\text{rise}}{\text{run}}$

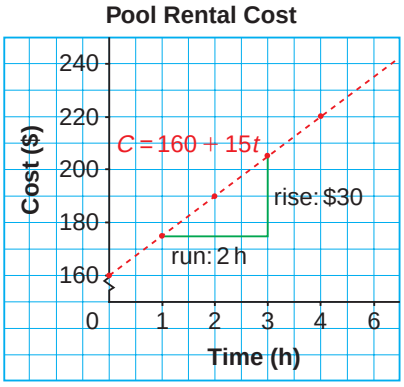
The equation is: $C = 8t$

rate of change is $8/h

Use Frayer models to show what you know about direct variation and partial variation.

➢ Partial Variation

A graph that represents partial variation is a straight line that does not pass through the origin.



Time t (h)	Cost C ($)	First Differences
0	160	
1	175	$175 - 160 = 15$
2	190	$190 - 175 = 15$
3	205	$205 - 190 = 15$
4	220	$220 - 205 = 15$

Rate of change $= \frac{\text{rise}}{\text{run}}$

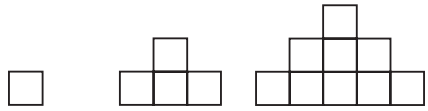
fixed cost variable cost

The equation is: $C = 160 + 15t$

vertical intercept is $160 rate of change is $15/h

What Should I Be Able to Do?

6.1 **1.** This pattern continues.



a) Sketch the next frame in the pattern.

b) Copy and complete the table below for the first 6 frames.

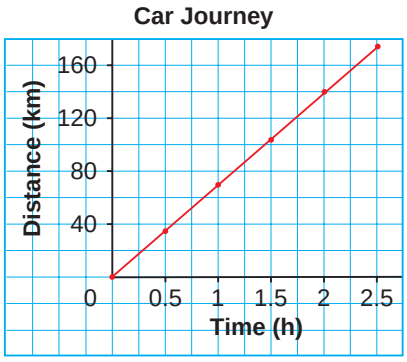
Frame number	Number of squares	First differences
1		
2		
3		

c) Is the relationship linear or non-linear? Explain.

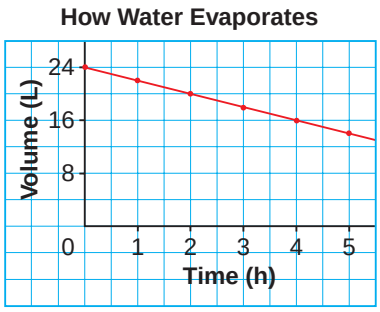
d) Graph the relationship. Does the graph support your answer to part c? Explain.

6.2 6.3 **2.** Determine each rate of change. Explain what it means.

a)



b)



6.4 6.5 **3.** Does each graph in question 2 represent direct variation or partial variation? How do you know?

6.4 **4.** Ken delivers packets of flyers for local stores. His pay varies directly as the number of packets delivered.

Number of packets delivered	0	100	200	300
Pay ($)	0	25	50	75

a) Graph *Pay* against *Number of packets delivered*.

b) Determine the pay for 250 packets delivered.

c) Suppose Ken wants to earn $100. How many packets must he deliver?

d) About how many packets does Ken have to deliver to earn $140?

6.5 **5.** The cost to rent a snowboard for an 8-h day is $28. If the snowboard is kept longer, there is an additional fee per hour.

Number of extra hours	0	1	2	3	4
Rental cost ($)	28	31	34	37	40

a) Graph *Rental cost* against *Number of extra hours*.

b) What does the vertical intercept represent?

c) What is the rate of change? What does it represent?

d) Write an equation to determine the rental cost, C dollars, for t extra hours.

6.6 **6.** To set up a hot air balloon, the crew inflates the balloon with cold air. One fan can blow 450 m^3 of air in 1 min.

a) Make a table of values for the volume, V cubic metres, of air in the balloon when the fan runs for up to 6 min.

b) Graph the data.

c) Write an equation to determine the volume V after t minutes.

d) Another fan can blow only half as much air per minute.

i) How does the graph change? Draw the new graph on the grid in part b.

ii) Write the new equation.

6.7 **7.** Solve each equation. Which tools could you use to help you?

a) $4x + 5 = 9$

b) $3 + 2x = -5$

c) $17 = 5 + 3x$

d) $18 = 3 - 5x$

e) $7x - 6 = 15$

f) $6x + 7 = -23$

6.8 **8.** A small pizza with tomato sauce and cheese costs \$6.00. Each additional topping costs \$0.75.

a) Write a rule for the cost of the pizza when you know how many additional toppings it has.

b) Write an equation for the cost, C dollars, of a small pizza when n toppings are ordered.

c) You have \$10.00 to spend. How many toppings can you order? Use two different methods to find out. Show your work.

6.9 **9.** The length of a person's foot is approximately 15% of his height.

a) What is the approximate foot length of a person who is 180 cm tall?

b) What is the approximate height of a person whose foot is 21 cm long?

c) How can you use an equation to answer these questions?

d) What other methods could you use to answer parts a and b?

6.10 **10.** Jo-Anne is choosing an Internet service provider.

- Speed Dot Company costs \$2.40 for every hour of Internet use.
- Communications Plus costs \$12 per month plus \$0.90 for each hour of Internet use.

a) Write an equation to model the cost for each billing system. Use C to represent the total cost, in dollars, and t to represent the time in hours.

b) Make a table of values up to 10 h for each relationship. Graph both relationships on the same grid.

c) What are the coordinates of the point of intersection? What do they represent?

d) Jo-Anne will use the Internet about 7 h per month. Which company should she choose? Explain your choice.

Practice Test

Multiple Choice: Choose the correct answer for questions 1 and 2.

1. Which equation represents partial variation?

A. $C = 20n$ **B.** $C = 20n^2$ **C.** $C = 20n + 5$ **D.** $C = -20n$

2. Which equation has the solution $x = 5$?

A. $2x + 6 = 7$ **B.** $2x + 5 = 9$ **C.** $2x + 4 = 11$ **D.** $2x + 3 = 13$

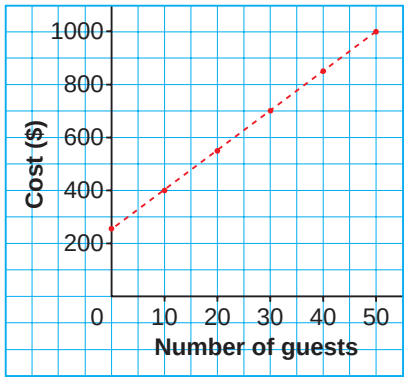
Show your work for questions 3 to 6.

3. Knowledge and Understanding

Andy's pay varies directly as the number of hours he works.
He earns $24 for working a 3-h shift.

a) Graph Andy's pay against time.
b) Determine the rate of change.
c) How much does Andy earn in 36 h? Explain.
d) How long does it take Andy to earn $216? Explain.

4. Communication The cost to rent a banquet hall is shown in the graph.
Write 5 different things you can determine from the graph.



5. Application

One ad in a local newspaper cost $70.
The ad had 25 words.
Another ad in the same newspaper cost $100.
It had 40 words.
How much would it cost to place an ad with 30 words?

a) Use a graph to solve the problem.
b) Use an equation to solve the problem.
c) Which did you prefer: using a graph or using an equation? Explain.

6. Thinking The school store wants to distribute brochures describing its products.
The store manager gets bids from two printers.
Printer A charges an initial fee of $280 plus $1 per brochure.
Printer B charges an initial fee of $90 plus $2 per brochure.
Which printer should the manager choose? Explain.