5 Graphing Relations

What You'll Learn

The relationship between two quantities can be illustrated with a graph that could be a straight line, a curve, or neither of these.

And Why

Relationships, such as the changes in temperature during a day or the time to run a certain distance, can be described with graphs. These graphs can be used to make predictions and to solve problems.

Key Words

- scatter plot
- trend
- line of best fit
- curve of best fit
- linear relation
- non-linear relation

Project Link

- Help Find the Stolen Mascot!

Reading and Writing in Math

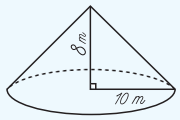
Writing Solutions

When your teacher asks you to "show your work," you must show all your thinking and write a complete solution to the question.

One question in a text was:

"A cone has height 8 m and base radius 10 m.
Determine the volume of the cone.
Show your work."

Here are 2 students' solutions.



Determine the volume of this cone.
radius of base: 10 m
height: 8 m
Formula: $V = \frac{1}{3}\pi r^2 h$
Substitute $r = 10$, $h = 8$
$V = \frac{1}{3}\pi(10)^2 \times 8$
$\doteq 837.76$
The volume of the cone is approximately 838 m^3.

$\frac{1}{3} \times \pi \times 100 \times 8$
$= 837.76$

- The solution on the left is more detailed. List some things that are included in the detailed solution.
- Suppose you did not know what the question was. Could you follow the thinking of the student on the right? Explain.
- Which solution looks more like your solutions?

Here are some tips to write a solution:

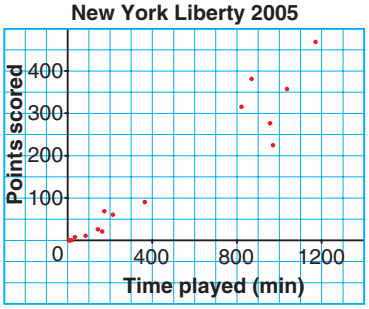
- Write the question.
- Show all your steps so someone else can follow your thinking.
- Include graphs, tables, or diagrams if they help explain your thinking.
- Is your solution reasonable? If it does not make sense, check all your calculations.
- Use math symbols, such as = and +, correctly.
- Include symbols, such as m and m^2, correctly.
- Write a sentence that answers the question.
- Include part of the question in the answer.

5.1 Interpreting Scatter Plots

The New York Liberty is a team in the Women's National Basketball Association (WNBA). Statistics for the players, such as points scored and time played, are recorded. Coaches can use these data to help them decide who should play.

Investigate Interpreting a Scatter Plot

During the 2005 regular season, 15 athletes played for the New York Liberty.
The scatter plot shows data for the team. Your teacher will give you a larger copy of this graph.



- One athlete played for 361 min.
  About how many points did she score?
  How many athletes scored fewer points than she did?
- Another athlete scored 353 points.
  About how many minutes did she play?
- How many athletes scored more than 200 points?
  Which of these athletes played for the fewest minutes?
- Suppose an athlete played many minutes but only scored a few points. In which region of the graph might her point be? Justify your answer.
- Is there a relationship between time played and number of points scored? Justify your answer.

Reflect

Suppose you are a basketball coach.
You select two of these players for your team.
Based on these data, which two players would you select?
Justify your choice.

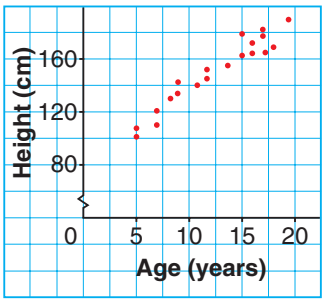
Connect the Ideas

A scatter plot is used to relate two different measures. For example, the scatter plot in *Investigate*, page 147, relates the points each player scored with the time she played.

A relationship between the measures is shown by a **trend** in the data points.

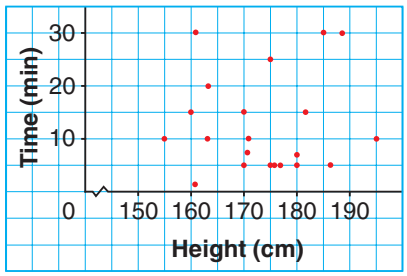
This mark, ⦚, on an axis indicates that numbers are missing from the scale.

Height and Age of Students



The points in this scatter plot show a relationship. As you move to the right, the points go up. There is an upward trend.

Height and Time to Travel to School for High School Students



The points in this scatter plot do not show a relationship. There is no trend in the data.

Practice

Your teacher will give you a large copy of each scatter plot.

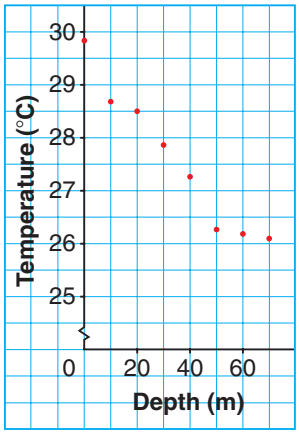
1. Lake Turkana is in the Great Rift Valley in northwest Kenya. As part of a study of water quality, the water temperature in the lake was measured at different depths. The data are shown in this scatter plot.

a) What was the approximate temperature at a depth of 70 m? At the surface?

b) At what depth was the temperature a little more than 27°C?

c) Do the points in the scatter plot show a relationship? Explain your thinking.

Water Temperature and Depth in Lake Turkana



A scatter plot can help you to identify trends in data.

Example

The scatter plot shows temperature data for some Ontario cities in November.

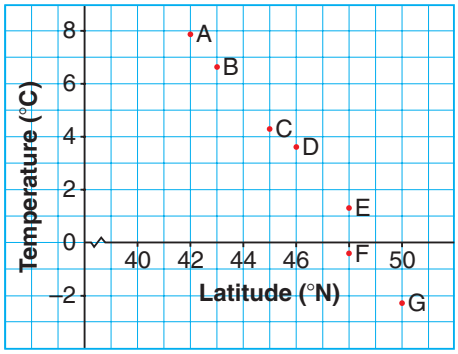
a) What does point D represent?

b) What do points E and F represent?

c) What does point G represent?

d) Is there a trend in the data? Explain.

Average Daily Maximum Temperature in November



Solution

a) Point D represents a city with an average daily maximum temperature of about 3.5°C. It is located at latitude 46°N.

b) Points E and F represent two cities with latitude 48°N. They have different average daily maximum temperatures.

c) Point G represents a city with latitude 50°N and an average daily maximum temperature of about −2°C.

d) There is a trend in the data.
As we move to the right along the *Latitude* axis, the points on the scatter plot go down.
That is, as the latitude increases, the temperature decreases.
This means that the farther north an Ontario city is, the lower the average daily maximum temperature is for November.

Latitude is a measure of how far north (or south) of the equator a city is.

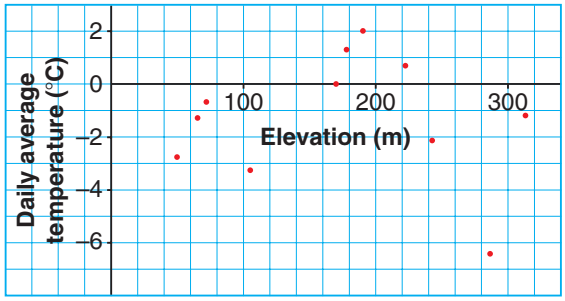
2. a) What does this scatter plot show?

b) One data point is for Windsor, with elevation 190 m. What is the average daily temperature in Windsor for March?

c) One data point is for Dunnville, with an average daily temperature of 0°C for March. What is Dunnville's approximate elevation? How do you know?

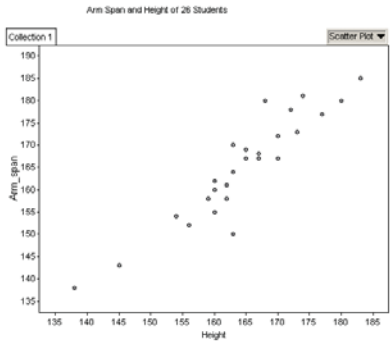
d) Describe any trends in the data. Justify your answer.

Average Daily March Temperature for Selected Ontario Cities and Towns



Elevation is the height of a city or town above sea level.

3. **Assessment Focus** The *Census at School* project collects data from high school students across Canada and around the world.
The graph shows data for 26 students.



Source: Statistics Canada, *Education Matters: Insights on Education, Learning, and Training in Canada*. "Census at School". Catalogue no. 81-004-XIE, Vol. 2, No. 5, released February 28, 2006. Date extracted June 23, 2006.

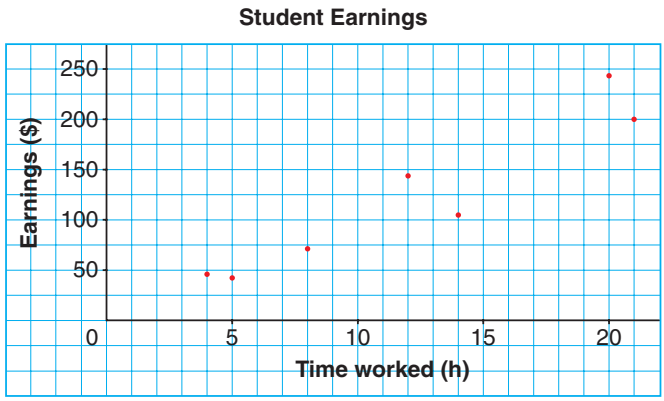
The arm span and height are measured in centimetres.

a) Liam is 154 cm tall.
   i) What is his approximate arm span?
   ii) How many students have an arm span shorter than Liam's?

b) Maya is 177 cm tall.
   i) What is her approximate arm span?
   ii) How many students have an arm span longer than Maya's?
   iii) Are any of these students taller than Maya? Justify your answer.

c) Describe any trends in the data. Justify your answer.

4. As part of a social studies project, a group of high school students compared their earnings from part-time jobs.



a) One student earned $112.
   i) How many hours did she work?
   ii) How many students earned more money than this student?
   iii) Did any of these students work fewer hours? Explain.

b) i) About how much did the student who earned the most make?
   ii) How many hours did this student work?
   iii) Did any students work more hours?

c) Describe any trends in the data. Justify your answer.

5. **Take It Further** Refer to the scatter plot in question 4.

a) One student worked 8 h.
   i) About how much did this student earn?
   ii) What is this student's hourly rate?

b) Identify one student who earned more per hour than the student in part a, and one who earned less. Explain how you know.

In Your Own Words

Describe how a scatter plot can be used to show a trend in data.
Use pictures, words, and numbers.

5.2 Line of Best Fit

To assess a person's nutritional needs, a doctor should know the person's height. But it can be difficult to measure the height of a person with a disability. Researchers look for other measures that can be used to estimate a person's height, such as knee height, arm length, and arm span.

Investigate Trends in Measurement

Do you think a taller person has longer arms than a shorter person? Answer the question, then conduct the following experiment to check your prediction.

Work in a group of 3.
You will need a measuring tape.
Work together to measure each person's height and arm length.
Record these measurements on the board.
Measure each person's knee height.
Record your knee height and your height in your notebook for use in *Practice*, question 5, page 155.

Copy the data from the board into a table like this.
Draw a scatter plot.
Does there appear to be a relationship between height and arm length?
How do you know?
Compare your results with your prediction.
If they are different, explain why.

Height (cm)	Arm length (cm)

Keep your scatter plot for use in *Practice*, question 2, page 153.

Reflect

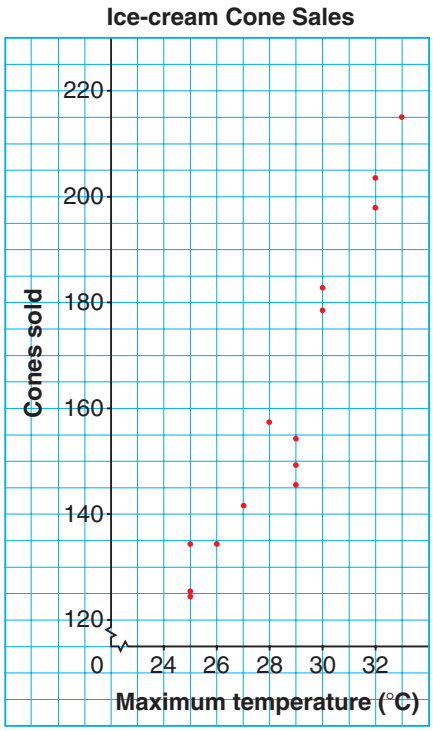
Use your scatter plot to answer these questions.
Justify your answers.

- A person is 160 cm tall. What might her arm length be?
- A person has arm length of 75 cm. How tall might he be?

Compare your answers with those of your classmates.
If the answers are different, explain why.

Connect the Ideas

The owner of an ice-cream stand wants to predict the number of ice-cream cones she will sell each day.
From her experience, she thinks the number may be related to the daily maximum temperature.
For 2 weeks, the owner records the maximum temperature and the number of cones sold each day.
She then draws a scatter plot.



Draw the scatter plot

Recall that this mark, ⦚, on an axis indicates that numbers are missing from the scale.

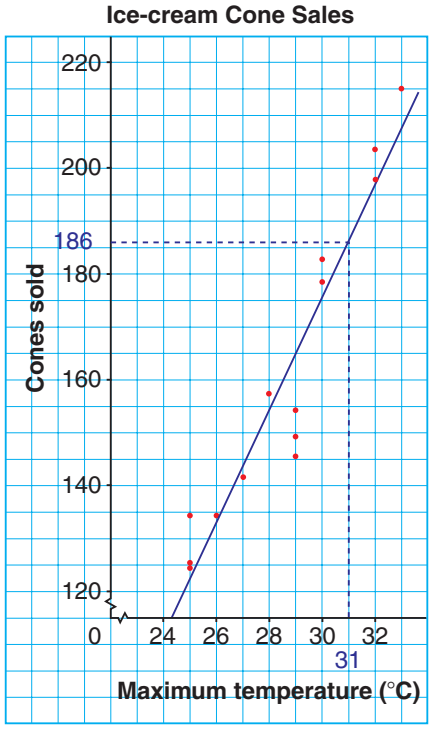
The scatter plot shows a trend in the data. The points go up to the right. That is, as the daily maximum temperature increases, more cones are sold.

Draw the line of best fit

There should be about as many points above the line as below.

To help predict the number of cones that might be sold, we draw a **line of best fit**.

To do this, place a ruler on the graph so that it follows the path of the points.
Draw a straight line along the edge of the ruler.



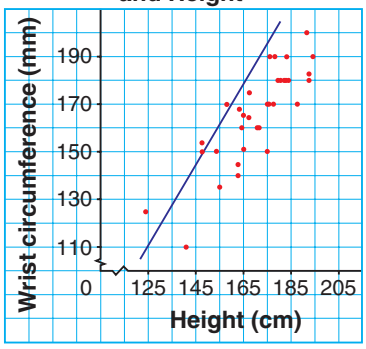
Use the line to estimate

One day, the predicted maximum temperature is 31°C.
To estimate the number of cones that might be sold, use the line of best fit.
Begin at 31 on the *Maximum temperature* axis.
Move up to the line of best fit, then over to the *Cones sold* axis.
When the temperature is 31°C, about 186 cones might be sold.

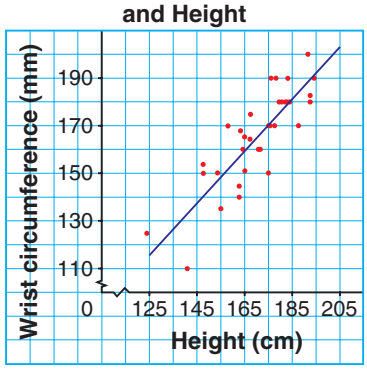
Practice

1. Adrianna drew a scatter plot and three different lines.
Which line would you use as a line of best fit? Justify your choice.

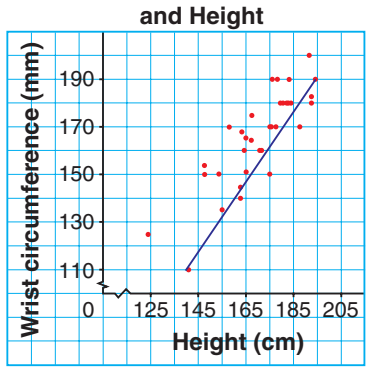
a)



b)



c)



2. Use the scatter plot you drew in *Investigate*, page 151.

a) Draw a line of best fit.

b) Use your line to answer the two questions in *Reflect*, page 151.

c) How do your answers in part b compare to your predictions in *Reflect*?

d) Compare your predictions with those of two classmates.
Does the line of best fit give closer predictions than the graph without the line? Explain.

3. Ask your teacher for a copy of the two scatter plots from Section 5.1, page 149.

a) Only one of these scatter plots may be modelled with a line of best fit.
Which is it? Explain why you should not draw a line of best fit for the other scatter plot.

b) Draw a line of best fit for the plot you chose in part a.
Use your line to make a temperature prediction for a latitude or elevation not included on the scatter plot.

You can extend a line of best fit to predict values beyond the data points.

Example

The table shows the world record times for women's 500-m speed skating from 1983 to 2001.

Year	1983	1986	1987	1987	1988	1994	1995	1997	1997	1997	2001	2001	2001
Time (s)	39.69	39.52	39.43	39.39	39.10	38.99	38.69	37.90	37.71	37.55	37.40	37.29	37.22

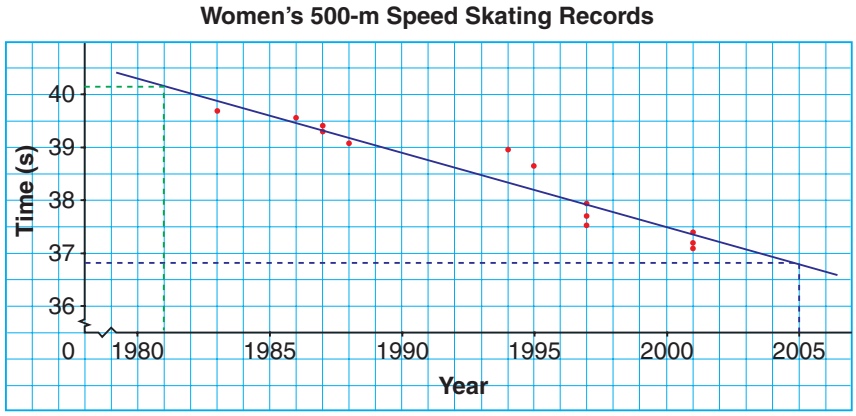
a) What trend do you see in the data?

b) Draw a scatter plot and a line of best fit.

c) What might the record have been in 1981?
d) When might the record time be less than 37 s?
e) What assumptions are you making in part d?

Solution a) The world record times are decreasing.
That is, the skaters are getting faster.

b)



c) From the line of best fit, the world record in 1981 might have been about 40.2 s.
d) From the line of best fit, the world record might drop below 37 s by 2005.
e) We assume the skaters can keep improving their times.

4. A ball is dropped from different heights.
The drop height and rebound height are recorded.

Drop height (m)	Rebound height (m)
1.0	0.7
2.0	1.3
3.0	2.3
4.0	3.0
5.0	3.8

a) What trend do you see in the data?
Does the rebound height of the ball appear to depend on the height from which the ball is dropped?
b) Draw a scatter plot.
Does the scatter plot support your answers to part a? Explain.
c) Draw a line of best fit.
d) Predict the rebound height when the ball is dropped from a height of 2.5 m and from a height of 8 m.
How did you do this?
e) Write one other question about these data.
Answer your question.
Show your work.

5. **Assessment Focus** In *Investigate*, page 151, you recorded your height and knee height.
   a) Do you think a person's knee height is related to her or his height? Explain.
   b) Use your measurements and those of 15 classmates.
      Draw a scatter plot.
      If possible, draw a line of best fit.
      Do the data support your answer to part a? Explain.
   c) Estimate the knee height of a person who is 180 cm tall.
   d) Estimate the height of a person with a knee height of 50 cm.

6. Is a person's shoe size related to her or his height?
   To check your answer, use the height data from *Investigate*, page 151, and collect shoe size data.
   a) Graph the data. Can you draw a line of best fit? Justify your answer.
   b) Do the data support your answer? Explain.

7. Choose a topic to investigate that involves two measures.
   Pose a question about the relationship between the measures.
   Conduct an experiment or research to find the data you need to answer the question.
   Present your solution using words, numbers, and graphs.
   If the data appear to lie along a line, include a line of best fit on the graph.

8. Will two people always draw the same line of best fit for a set of data? Explain.

9. **Take It Further** Conduct research to find the world record times for men's 500-m speed skating from 1983 to 2001.
   a) What trend do you see in the data?
   b) Draw a scatter plot and a line of best fit.
   c) Estimate what the record might have been in 1981.
      How did you do this? Do some research to check the estimate.
   d) Write a question that can be answered using the line of best fit.
      Prepare a solution for the question.
      Give the graph and question to a classmate to answer.
      Discuss and compare your solutions.

In Your Own Words

List 2 reasons you might draw a line of best fit after plotting data.
Provide an example for each reason.

Using *Fathom* to Draw a Line of Best Fit

Technology

As part of the international project called *Census at School*, Statistics Canada has collected data on Canadian high school students.
We will use *Fathom* to explore relationships in the data for 104 Ontario students. These students were selected from the *Census at School* database.

Here is a "collection inspector" window for the data set:

In *Fathom*, a collection is a data set.

Inspect Ontario High School Stud...

Cases | Measures | Comments | Display

Attribute	Value
Gender	Male
Age	15
Height	179
Hand_Span	23
Wrist	151
Foot	28
Eyes	brown
Postal_code	L7M
Household_Size	5
Travel_to_school	car
time_to_school	11
Favourite_subject	physical education
Look_up_to	religious figure
Plans	university

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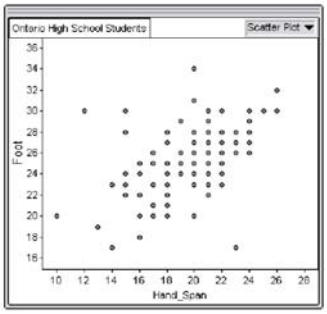
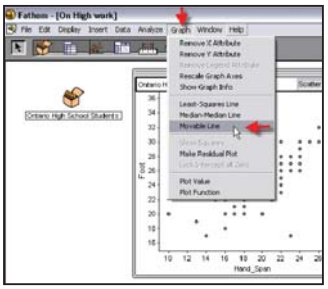
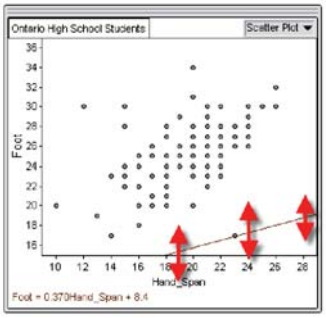
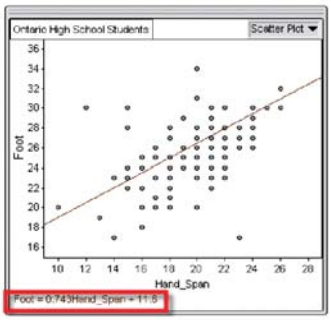
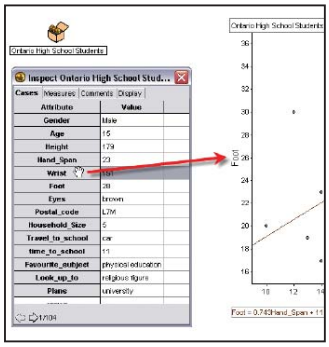
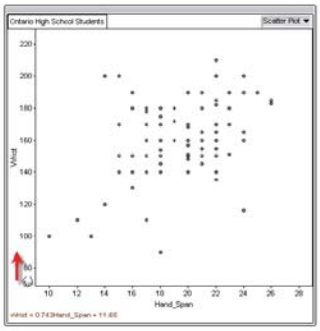
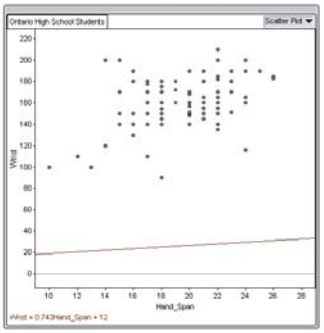
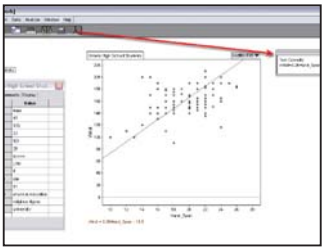
Source: Statistics Canada, *Education Matters: Insights on Education, Learning, and Training in Canada*. "Census at School". Catalogue no. 81-004-XIE, Vol. 2, No. 5, released February 28, 2006. Date extracted June 23, 2006.

We see that this person is a 15-year-old male who is 179 cm tall, has a hand span of 23 cm, a wrist circumference of 151 mm, a foot length of 28 cm, and brown eyes.
We can also see information about his home and his life.
Notice that some attributes, such as height, are described by a number.
Other attributes, such as plans, are described in words.

We will create scatter plots to help us look for relationships between attributes described by numbers.

Both the hand span and foot length are measured in centimetres.

Use *Fathom* to open the file *Ontario High School Students 1*. This is a ready-made data set on 104 students.	
You will see a scatter plot showing hand span on the horizontal axis and foot length on the vertical axis. We see that, generally, the greater a person's hand span, the longer her or his foot.	
To model this relationship with a line: Click on the graph so it becomes active. The **Graph** menu appears on the toolbar at the top of the screen. Select the **Graph** menu. Then select **Movable Line**.	
"Fit" the brown line to the data so it illustrates the relationship between foot length and hand span. You can "grab and drag" the ends of the brown line to make it steeper or less steep. You can "grab and drag" the middle of the brown line to move it up or down.	
When the line fits the data the best, the data points should be equally distributed on both sides of your line of best fit. Notice that *Fathom* gives you the equation for your line of best fit. Print the graph.	

Double-click the collection. The first case in the collection is displayed beside the graph.	
To explore if there is a relationship between hand span and wrist circumference: Left click, drag, and drop the "Wrist" attribute from the collection inspector to the vertical axis of your scatter plot. The foot length data are replaced by the wrist circumference data.	
Because the vertical axis now begins at a greater number, the brown movable line is no longer visible. "Grab" the bottom of the vertical axis and drag it up, until the movable line appears. "Grab and move up" the line. You can "grab and push" the vertical axis back where it was.	
"Fit" the movable line to the data. Describe the relationship between hand span and wrist circumference. Do you think it is appropriate to draw a line of best fit for these data? Justify your answer. Print your graph.	
Insert a text box. Type in your name. Save your file. Try to find relationships between other attributes described by numbers.	

For each relationship you investigated, compare your line of best fit with those of other students. How are they similar?

5.3 Curve of Best Fit

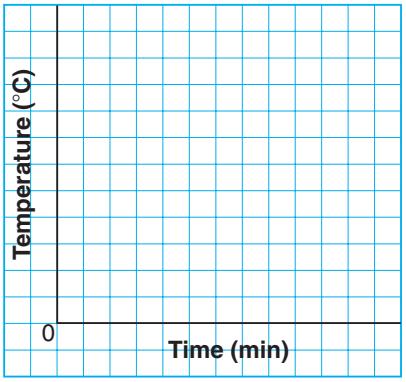
Investigate Relationship between Temperature and Time

Work in a group of 3.
You will need: a plastic cup, an insulated cup, crushed ice, water, a measuring cup, and a thermometer

Place equal volumes of ice and warm water in each cup.
Predict what will happen to the ice and water in each cup.
Predict how the temperature will change over time.
Give reasons for your predictions.

Measure the temperature of the contents of each cup at regular intervals for 30 min.
How often do you think you should measure the temperatures? Why?
Record the times and temperatures in a table.

Use a different colour to plot each set of data on the same grid.



How are the graphs the same?
How are they different?
Describe any trends in the graphs.
Do the results support your predictions? Explain.

How long did it take the contents of each cup to reach a constant temperature?

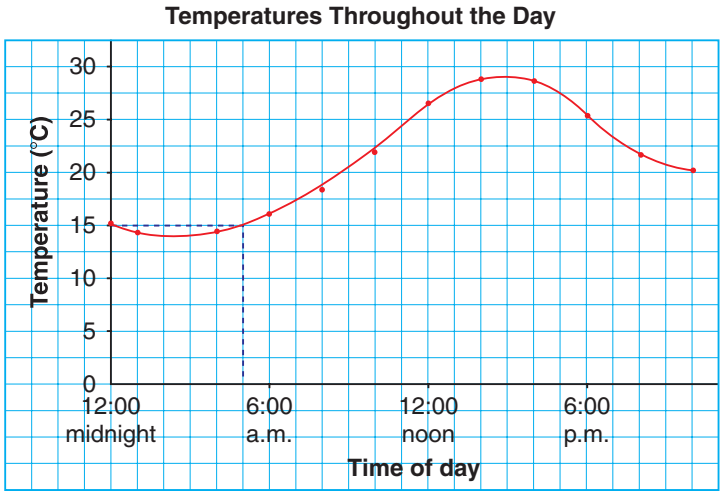
Reflect

- What factors might affect the results of this experiment?
- How could you change this experiment to account for these factors?

Compare your results with those of your classmates.
If they are different, explain why.

Connect the Ideas

A weather forecaster measures and plots the temperature every 2 h on a summer day.
The temperatures decrease from midnight to 4:00 a.m., increase once the sun rises, reach a maximum in the afternoon, then decrease again during the late afternoon and evening.
The points do not lie on a straight line, but appear to be related.
These data can be approximated by a curve.
We call it a **curve of best fit**.
To draw a curve of best fit, draw the smooth curve that passes through as many points as possible.

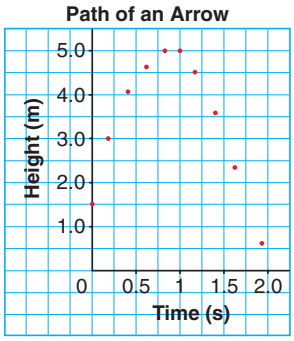


The greatest temperature occurred between 2:00 p.m. and 3:00 p.m.
We can use the curve to estimate the temperature at 5:00 a.m.
Begin at 5:00 a.m. on the *Time of day* axis.
Move up to the curve, then across to the *Temperature* axis.
At 5:00 a.m., the temperature was about 15°C.

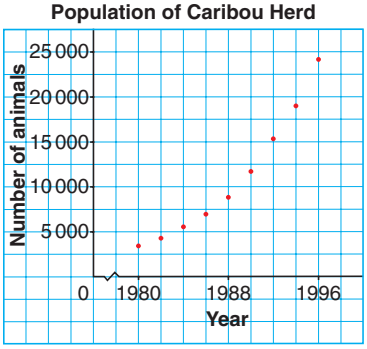
Practice

1. Your teacher will give you a copy of each graph.
Describe any trends in the data. Draw a curve of best fit for each set of data.

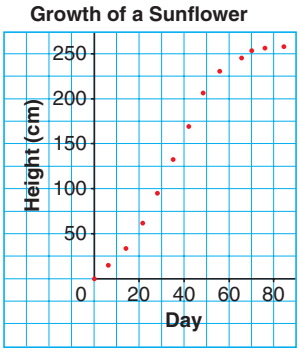
a)



b)



c)



2. The table shows the number of hours of daylight in Waterloo, Ontario, for the first day of each month in 2005.

Month	Jan	Feb	Mar	April	May	June	July	Aug	Sep	Oct	Nov	Dec
Daylight hours	9.0	9.9	11.2	12.8	14.2	15.2	15.4	14.5	13.2	11.7	10.3	9.2

a) What trend do you see in the data? Explain the trend.
b) Graph the data. Draw a curve of best fit.
c) Estimate the number of hours of daylight on March 15.
d) The day with the most daylight is June 21.
Estimate the number of hours of daylight on June 21.
e) Estimate the number of hours of daylight on your birthday.
How did you do this?

You have used a curve of best fit to predict values that lie between data points.
You can also extend a curve of best fit to predict values beyond the data points.

Example

A soccer ball is kicked up into the air from the ground.
The height of the ball is measured at regular time intervals.
Here are the data.

Time (s)	0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6
Height (m)	0	2.2	4.0	5.4	6.5	7.1	7.3	7.2	6.7

a) What trend do you see in the data? Explain the trend.
b) Graph the data. Draw a curve of best fit.
c) When do you think the ball is at its greatest height?
Use the graph to check.

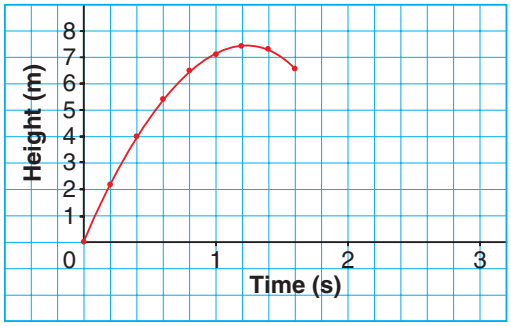
d) When is the ball 5 m high?

e) When does the ball hit the ground? How does the graph show this?

Solution

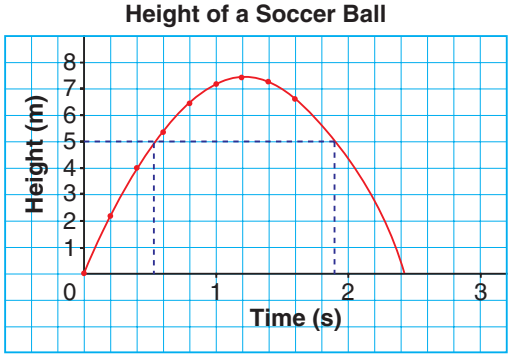
a) The height of the ball increases, then decreases.
That is, the ball stops rising and begins to fall.

b)



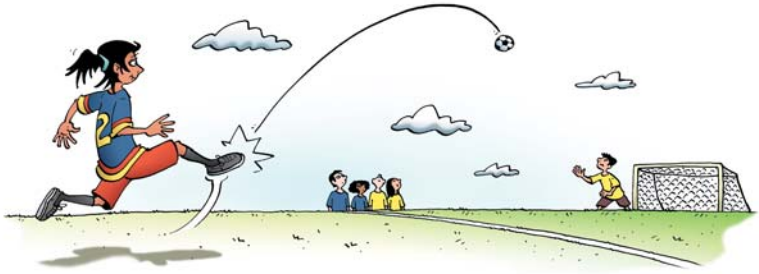
c) From the table and graph, the greatest height of the ball seems to occur at 1.2 s when the ball is 7.3 m high.

d) There are two times when the ball is 5 m high:
once as it is rising and once as it is falling.
To determine the second time, extend the graph to the right.
Draw a smooth curve that "mirrors" the curve from the starting point to its greatest height.



From the graph, the ball is 5 m high at approximately 0.5 s and 1.9 s.

e) The extended curve shows that the ball hits the ground after about 2.4 s, when the height is 0 m.
This is where the curve meets the *Time* axis.



3. An Internet host is a computer directly connected to the Internet. The number of Internet hosts around the world has grown quickly. The data in the table are for January of each given year.

Year	1992	1994	1996	1998	2000	2002	2004	2006
Internet hosts (millions)	0.7	2.2	9.5	29.7	72.4	147.3	233.1	395.0

a) What trend do you see in the data? Explain the trend.
b) Graph the data. Draw a curve of best fit.
c) Estimate the number of Internet hosts in 2001.
d) When might the number of Internet hosts reach 500 million? Justify your answer.

4. **Assessment Focus** The high divers at Paramount Canada's Wonderland perform competitive dives from a height of 21 m. A diver's height is measured every 0.2 s.

Time (s)	0	0.2	0.4	0.6	0.8	1.0	1.2
Height (m)	21	20.7	20.2	19.4	17.6	16	14

a) What trend do you see in the data? Explain the trend.
b) Graph the data. Draw a curve of best fit.
c) Estimate when the diver will be 10 m above the water.
d) When does the diver reach the pool? How do you know?

5. **Take It Further** The table shows the number of people enrolled in apprenticeship programs in Canada, rounded to the nearest thousand.

Year	1995	1996	1997	1998	1999	2000	2001	2002
Females (thousands)	11	12	13	14	16	17	20	22
Males (thousands)	153	154	159	163	173	184	198	213

a) Graph both sets of data on one grid.
b) For each data set, draw a line or curve of best fit. How did you decide which to draw?
c) Estimate the numbers of females and males enrolled in apprenticeship programs in 2003. How did you do this?

In Your Own Words

Suppose you graph data for which a curve of best fit can be drawn. How do you decide where to draw the curve? Include a graph in your explanation.

Using a Graphing Calculator to Draw a Curve of Best Fit

Technology

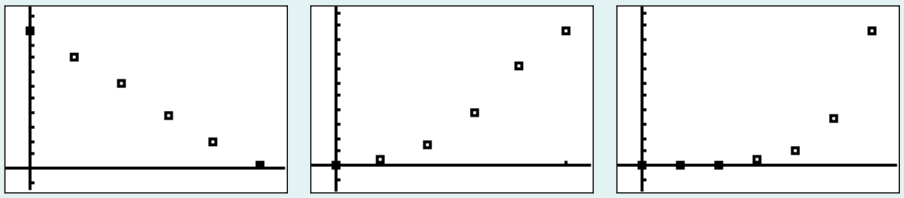
When data points appear to lie along a curve, we say the relation is **non-linear**.
There are many mathematical models to describe non-linear relations.
Two models are a quadratic curve of best fit and an exponential curve of best fit.

The TI-83 and TI-84 graphing calculators can model a relation with

- a line of best fit (**LinReg**)
- a quadratic curve of best fit (**QuadReg**)
- an exponential curve of best fit (**ExpReg**)

Three sample scatter plots are shown.
Each plot represents a different type of relation.

It can be difficult to distinguish between quadratic and exponential relations. The quadratic curve increases more gradually.



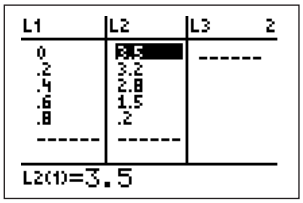
Linear Quadratic Exponential

A ball is dropped from a height of 3.5 m.
The height of the ball as it drops is measured using a CBR motion detector.
The data are shown in this table.

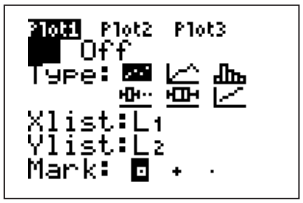
Time (s)	0.0	0.2	0.4	0.6	0.8
Height (m)	3.5	3.2	2.8	1.5	0.2

Use a TI-83 or TI-84.
Follow these steps to model this relation.

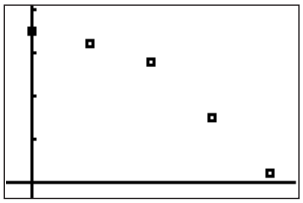
Press STAT 1 to access the List Editor. If necessary, clear the lists L1 and L2. To do this, move the cursor to the column head and press CLEAR ENTER. Enter the times in L1, and the corresponding heights in L2. After you key in each number, press ENTER. Before graphing the data, make sure there are no equations in the Y= list. To do this, press Y=. Move the cursor to any equation, then press CLEAR.



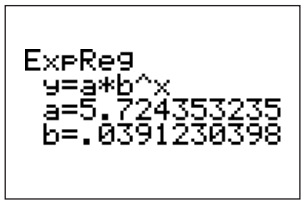
Press 2nd Y= 1 to access the Stat Plot Editor for Plot 1. Set the plot options as shown here. Then press ZOOM 9. This instructs the calculator to set the window so all the data can be displayed, and graphs the data.



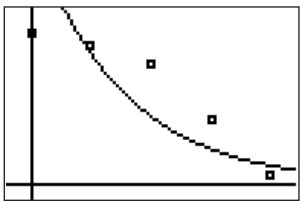
The graph should look like the one shown here. The points do not lie on a line. We will model the data with an exponential curve and a quadratic curve, and decide which is the better fit.

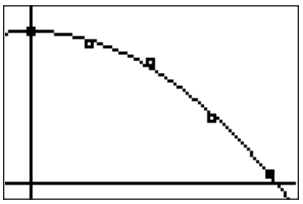


To draw an exponential curve of best fit, press STAT ▶ 0 ENTER. The calculator displays the equation of an exponential curve that models the data.



To show the curve, press Y= CLEAR. Press Y=. Then press VARS 5 ▶ ▶ 1 GRAPH.



To draw a quadratic curve of best fit, press STAT ▶ 5 ENTER. The calculator displays the equation of a quadratic curve that models the data.	QuadReg y=ax²+bx+c a=-5.178571429 b=-.0071428571 c=3.485714286
To show the curve, press Y= CLEAR. Then press VARS 5 ▶ ▶ 1 GRAPH.	
Which model better represents the data? Justify your choice.	

Complete these steps for each set of data below.

Use pages 165 and 166 as a guide.

- ➢ Use a TI-83 or TI-84 to graph the data.
- ➢ Draw an exponential curve of best fit and a quadratic curve of best fit. Sketch each curve.
- ➢ Decide which curve better models the data. Justify your answer.

1. In 2002, movie attendance in Canada hit a 44-year high. The table shows the attendance in several other years for comparison.

Year	1991	1998	1999	2000	2002
Movie attendance in Canada (millions)	69.2	109.7	117.4	117.6	124.2

2. In ideal conditions, *E. coli* bacteria cells divide every 20 min. The table shows how the number of cells in a sample would grow over time.

Time (min)	0	20	40	60	80	100	120
Number of cells	1	2	4	8	16	32	64

Hidden Sum

GAME

Play in a group of 3.
You will need a calculator.
As a group, choose a target number greater than 200.

The first player enters a number into the calculator,
presses the Memory Plus key [M+],
and passes the calculator to the second player.

The next player enters a number and presses the [M+] key.
The number is added to the number already stored in the memory.

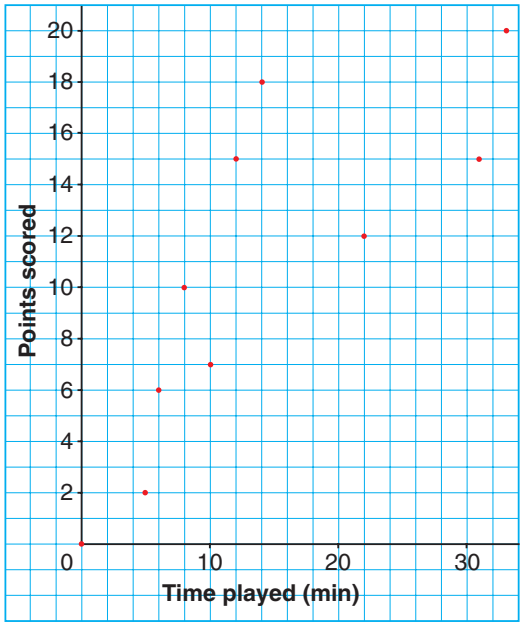
Take turns entering numbers and pressing the [M+] key.

Each number entered must be less than 50.

When you think the sum is equal to or greater than
the target number, call out "Over!"
Use the Memory Recall key [MR] to check.
If you are correct, you win the game.
If not, you are out of the game.
The other players continue taking turns.

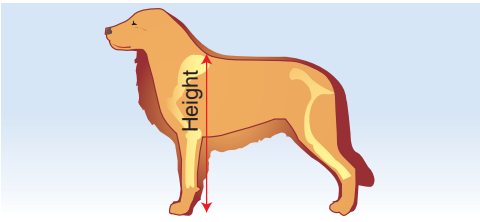
Mid-Chapter Review

5.1 **1.** Jordan and Esau compared the statistics of the junior boys' basketball team. They drew a scatter plot.



a) What does each point represent?
b) How many points were scored by the boy who played 10 min?
c) How many boys scored 15 or fewer points? More than 15 points?
d) Suppose a boy played few minutes but scored many points. Where would his point on the graph be?
e) Describe any trends in the data. Justify your answer.

5.2 **2.** Saskia and Maria are working on a science fair project. They investigate whether a dog's mass is related to its height.



Here are some data Saskia and Maria collected.

Height (cm)	69	59	54	46
Mass (kg)	35	32	23	17

Height (cm)	38	31	28	27
Mass (kg)	14	6	8	5

a) Describe any trends in the data.
b) Graph the data.
Draw a line of best fit.
c) Does there appear to be a relationship between a dog's height and mass?
Justify your answer.
d) Estimate the height of a dog with mass 16 kg.
e) Estimate the mass of a dog that is 50 cm high.
f) What assumptions did you make in parts d and e?

5.3 **3.** This table shows the world population every 10 years from 1900 to 2000. The populations are rounded.

Year	1900	1910	1920	1930	1940	1950	1960	1970	1980	1990	2000
Population (billions)	1.6	1.8	1.9	2.1	2.3	2.6	3.0	3.7	4.5	5.3	6.1

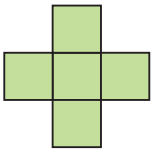
a) Describe any trends in the data.
b) Graph the data.
Draw a curve of best fit.
c) Estimate the population in 1925 and in 1975.
d) Estimate the population in 1890.
e) Predict the population in 2010.
f) What assumptions did you make in parts c, d, and e?

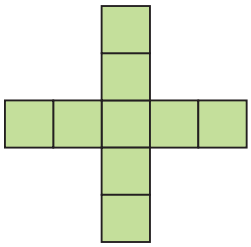
5.4 Graphing Linear Relations

Investigate Relationships in Patterns

Work with a partner.
You will need congruent square tiles and grid paper.


Frame 1


Frame 2


Frame 3

The first 3 frames in a pattern are shown at the left.

- Draw the frames on grid paper. Describe the pattern.
- Use tiles to construct the next 3 frames in the pattern. Draw the frames on grid paper.
- Record the number of tiles in each frame in a table like the one below.

Frame number	Number of tiles in a frame

Describe any trends in the data.

- Graph the data.
  Describe how you can use the graph to determine the number of tiles in a frame when you know the frame number.
  How many tiles will be in the 15th frame? The 22nd frame? The 100th frame?

Reflect

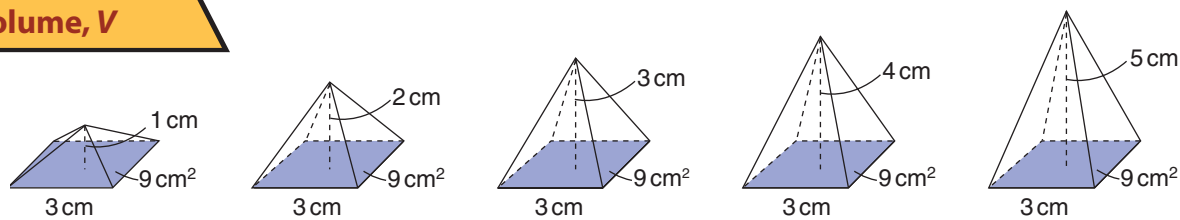
Write a rule to determine the number of tiles in any frame.
Check that your rule works.
Compare your rule with those of other students.
If the rules are different, explain why.

Connect the Ideas

Recall that volume of a pyramid: $V = \frac{1}{3}Bh$

We can explore how the volume of a pyramid is related to its height.
Here are several square pyramids with base area 9 cm^2.

Calculate the volume, *V*



$V = \frac{1}{3} \times 9 \times 1$
$= 3$

$V = \frac{1}{3} \times 9 \times 2$
$= 6$

$V = \frac{1}{3} \times 9 \times 3$
$= 9$

$V = \frac{1}{3} \times 9 \times 4$
$= 12$

$V = \frac{1}{3} \times 9 \times 5$
$= 15$

Graph the data

Here are the data and a graph.

Height (cm)	Volume (cm³)
1	3
2	6
3	9
4	12
5	15

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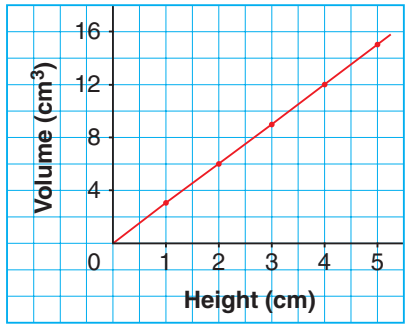
+ 3 + 3 + 3 + 3

Volumes of Square Pyramids with Base Area 9 cm^2



As the height increases by 1 cm, the volume increases by 3 cm^3.

We can join the points with a line since pyramids exist with heights between the heights in the table.

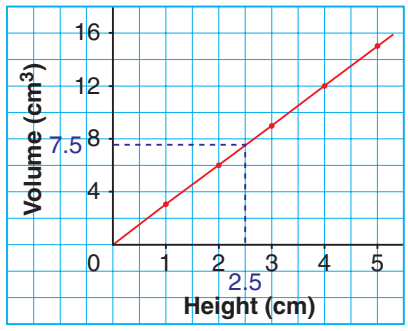
Since the graph is a straight line, we say there is a **linear relation** between the volume and height.

Use the graph to estimate

To estimate the volume of a pyramid with base area 9 cm^2 and height 2.5 cm:
Begin at 2.5 on the *Height* axis.
Move up to the line, then across to the *Volume* axis.

The volume is about 7.5 cm^3.

Volumes of Square Pyramids with Base Area 9 cm^2



Use a rule

From the table, for these pyramids,
the volume is 3 times the height.
We can calculate any volume using this rule.
When the height is 2.5, the volume is:

$V = 3 \times 2.5$
$= 7.5$

The volume of a pyramid with base area 9 cm^2 and height 2.5 cm is 7.5 cm^3.

The rule gives an accurate answer.

Practice

1. This graph shows how the Celsius and Fahrenheit temperature scales are related. Your teacher will give you a large copy of this graph.

a) Is the relationship linear? How do you know?

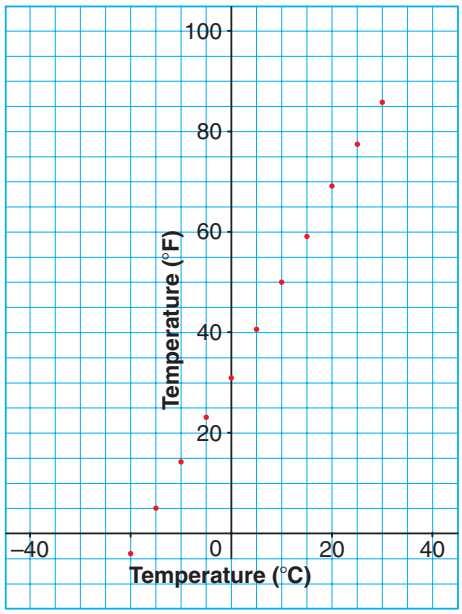
b) Use the graph to estimate the Fahrenheit equivalent of 25°C.

c) Use the graph to estimate the Celsius equivalent of –5°F.

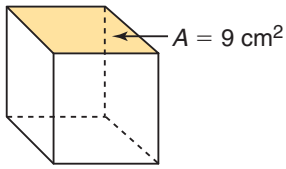
d) How could you check your answers to parts b and c?

e) Write a question that could be answered using the graph.
Exchange questions with a classmate.
Check each other's work.

Relating the Celsius and Fahrenheit Temperature Scales



2. In *Connect the Ideas*, page 170, you learned that there is a linear relationship between the volume of a square pyramid with base area 9 cm^2 and its height. Suppose we have a square prism with base area 9 cm^2 instead.



a) Do you think the relationship between volume and height is linear? Justify your answer.

b) Create a table of values for the volumes of prisms with heights from 1 cm to 5 cm.

c) Graph the data.

d) What trend do you see in the data? Is the relationship linear?

e) Write a rule for the relationship.

f) Determine the volume of a prism with base area 9 cm^2 and height 3.5 cm.
How many different ways could you do this? Explain.

Need Help?
Read Connect the Ideas.

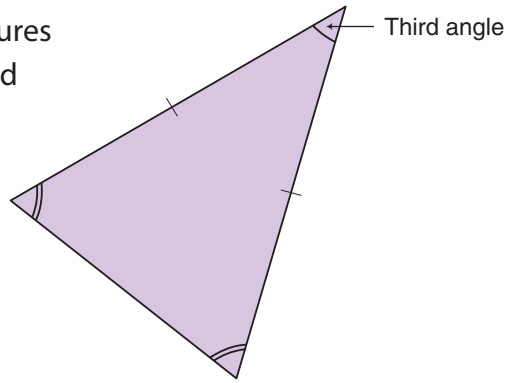
We can apply what we know about linear relations to geometry.

Example

How are the angles in an isosceles triangle related?

a) Make a table to show the measures of the equal angles and the third angle in an isosceles triangle. What trend do you see in the data?

b) Graph the data. Is the relationship linear? Justify your answer.



Solution

a) The sum of the angles in a triangle is 180°.
Add the two equal angles, then subtract the sum from 180° to determine the third angle.

Equal angle measure (°)	Third angle measure (°)
10	160
20	140
30	120
40	100
50	80
60	60
70	40
80	20

In Chapter 3, you learned how the angles in a triangle are related.

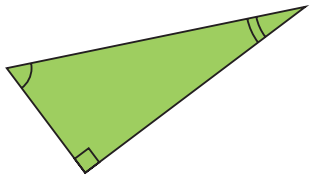
For example, when the equal angles are 10°, their sum is 20°, and the third angle is $180° - 20° = 160°$

From the table, we see that for every 10° increase in the equal angles, there is a 20° decrease in the third angle.

b) The points lie on a straight line. So, the relationship is linear.

Since we can draw isosceles triangles with angles between those in the table, we join the points with a line.

3. Investigate the relationship between the 2 acute angles in a right triangle.



a) Create a table of values.
Use 5°, 10°, 15°, 20°, and 25° as the measures of one angle.
Graph the data.

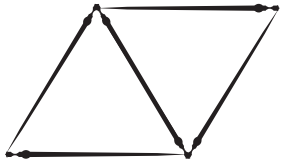
b) What trend do you see in the data and graph?
Is the relationship linear?
Justify your answer.

c) What is the measure of the second acute angle in a right triangle with one angle measuring 30°?
With one angle measuring 40°?
Show your work.

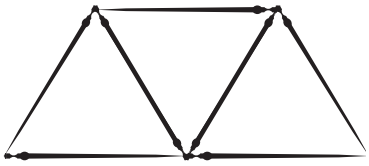
Need Help?
Read the Example on page 172.

4. **Assessment Focus** The first 3 frames in a pattern are shown at the right.
Use toothpicks to construct the frames.

Frame 1



Frame 2



Frame 3

a) Construct the next 3 frames in the pattern.
Record each frame number and the number of toothpicks in a table.

b) Do you think this is a linear relation? Explain.

c) Graph the data. Is the relationship linear?
Justify your answer.

d) Predict how many toothpicks will be in the 8th frame.
Justify your prediction.

e) Write a rule for the number of toothpicks in any frame.

f) Determine the number of toothpicks in the 15th frame.
How many different ways could you do this? Explain.

5. In Chapter 2, you drew different rectangles with the same perimeter. Here are the dimensions of some rectangles with perimeter 24 cm.

Width (cm)	Length (cm)
1	11
2	10
3	9
4	8
5	7
6	6

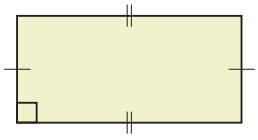
a) Is the relationship between length and width linear?
Justify your answer.

b) Graph the data.
Does the graph illustrate your answer to part a? Explain.

c) Write a rule for the relationship.

d) i) Determine the length when the width is 4.5 cm.
ii) Determine the width when the length is 10.5 cm.

e) How many different ways could you answer part d? Explain.

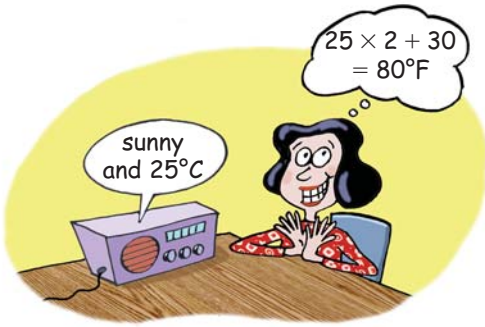


$P = 24$ cm

6. An approximate rule for converting Celsius temperature to Fahrenheit temperature is: Double the Celsius temperature and add 30.

a) Create a table of values for this rule.
Extend the table to 30°C.

Celsius temperature (°C)	Fahrenheit temperature (°F)
−20	
−15	
−10	

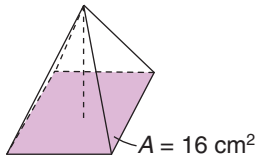


b) Graph the data.

c) Plot the line from the graph in question 1 on your graph.

d) Compare the two lines.
For which range of temperatures is the rule "double and add 30" useful?
Justify your answer.

7. **Take It Further** Investigate the relationship between the volumes and heights of square pyramids with base area 16 cm^2.



a) Create a table of values for pyramids with heights from 1 cm to 5 cm.

b) Graph the data.

c) What trend do you see in the data and the graph?
Is the relationship linear?
Justify your answer.

d) Estimate the volume of a pyramid with base area 16 cm^2 and height 7.5 cm.
How did you do this?

e) Estimate the height of a pyramid with base area 16 cm^2 and volume 60 cm^3.
How did you do this?

f) Write a rule.
How could you use the rule to check your answers to parts d and e?

In Your Own Words

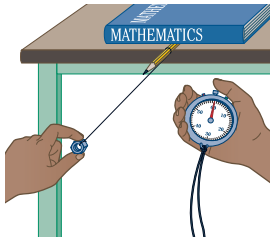
What patterns in data and in a graph show a linear relation?
Include examples in your explanation.

5.5 Graphing Non-Linear Relations

Investigate Motion of a Pendulum

Work with a partner.
Do you think a longer pendulum takes more time to complete 6 swings than a shorter pendulum?
You will experiment to check your prediction.

You will need a pencil, string, a nut, a heavy book, a metre stick, and a stopwatch. Make a pendulum as shown in the art.
It should be 50 cm long.



Set your pendulum swinging.
One swing occurs when the pendulum swings back and forth once. Let it swing a few times, then begin counting and timing.
Record the time it takes to complete 6 swings.
Repeat this process 3 more times.
Determine the mean time for the 4 trials.

To determine the mean, add the times then divide by 4.

Shorten the pendulum string to 40 cm and repeat the experiment.
Repeat the experiment 3 more times, shortening the length of the string to 30 cm, 20 cm, and 10 cm.
Copy and complete this table.

Pendulum length (cm)	Mean time for 6 swings (s)
50	
40	
30	
20	
10	

Describe any trends in the data.
Was your prediction correct?

Graph the data. Describe the graph. Then draw a curve of best fit.

Reflect

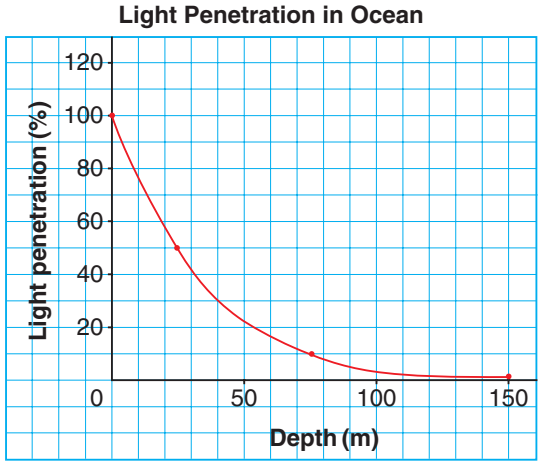
- How is the time for a pendulum to complete 6 swings related to the pendulum length?
- Suppose your pendulum was 25 cm long.
  Estimate how long it might take to complete 6 swings.
- What other factors do you think might affect the time for one swing?
  How could you check?

Connect the Ideas

Plankton and seaweed, which are the basis of ocean food chain, need sunlight to survive.
As sunlight enters the ocean, it is absorbed and scattered by the water.
The table and graph show how the amount of sunlight that penetrates clear tropical ocean water changes with depth.

Draw a curve of best fit

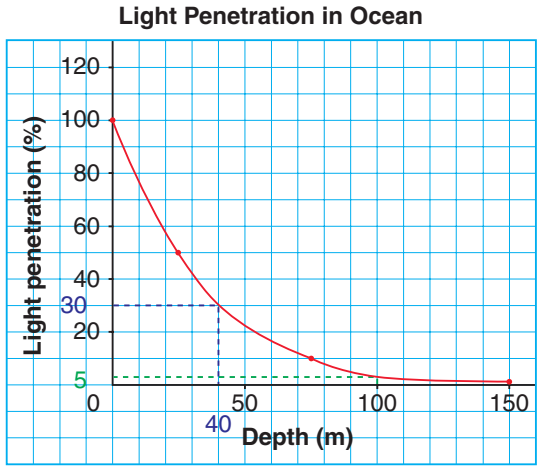
Depth (m)	Light penetration (%)
0	100
25	50
75	10
150	1



From the table and graph, we can see that the greater the depth, the less sunlight penetrates the water.
Since the points appear to lie on a curve, we draw a smooth curve to fit the data.

Use the curve to estimate

At a depth of 40 m, about 30% of sunlight penetrates the water.
At a depth of 100 m, only about 5% of sunlight penetrates the water.

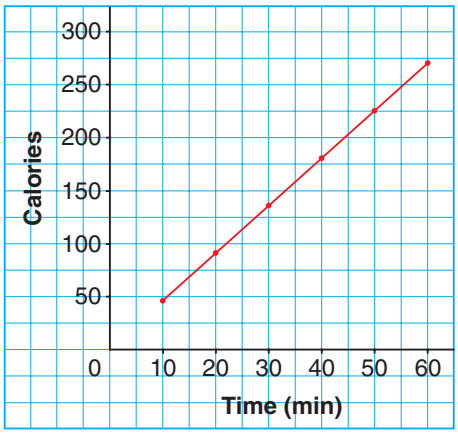


When the graph of a relationship is a curve, we say that the relationship is **non-linear**.

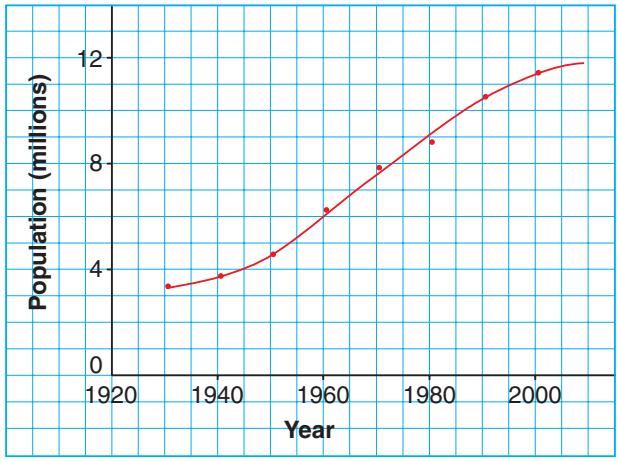
Practice

1. Two graphs are shown. Is each relationship linear or non-linear? Explain how you know.

a)



b)



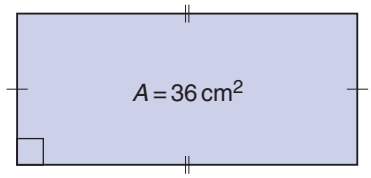
2. Use the graphs in question 1.

a) i) About how many calories does a 55-kg woman burn when she walks for 30 min? For 45 min?

ii) What was the approximate population of Ontario in 1945? In 1995?

b) Write a question of your own that can be answered using one of the graphs.
Exchange questions with a classmate. Answer your classmate's question.

3. In Chapter 2, you investigated the dimensions and perimeter of a rectangle when its area was given. Here are some data for a rectangle with area 36 cm^2.



Width (cm)	Length (cm)
1	36
2	18
3	12
4	9
6	6

a) Is the relationship between length and width non-linear? Justify your answer.

b) Graph the data.
Does the graph illustrate your answer to part a? Explain.

c) Use the graph.

i) Determine the length when the width is 5 cm.

ii) Determine the width when the length is 8 cm.

d) Write a rule for the relationship.

We can create a table of values from a description of a relationship.

Example

A typical North American adult consumes about 200 mg of caffeine a day.
Caffeine has a half-life of about 6 h.
This means that about 6 h after consumption, half the caffeine remains in a person's body.

Caffeine occurs naturally in coffee beans, cocoa beans, kola nuts, and tea leaves. It is also found in products, such as coffee, tea, some energy drinks and soft drinks.

a) Copy and complete the table below to show how much caffeine is left in a person's body over time.

b) What trends do you see in the data? What do you think the graph will look like?

c) Graph the data. Describe the graph. How does the graph compare to your prediction in part b?

d) About how much caffeine will remain after 9 h? After 36 h? What assumption did you make?

Time (h)	Mass of caffeine (mg)
0	200
6	
12	
18	
24	

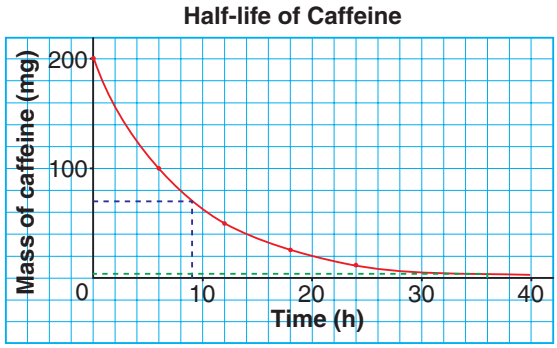
Solution

a) Every 6 h, the mass of caffeine is halved. The times in the table increase by 6 h each time. So, divide the mass by 2 to get the new mass each time.

Time (h)	Mass of caffeine (mg)
0	200
6	100
12	50
18	25
24	12.5

b) The mass of caffeine is decreasing. The amount by which the mass changes is also decreasing. So, the graph will be a curve.

c) Draw the curve of best fit through the points. The graph is a curve that goes down to the right.



d) From the graph, the mass of caffeine remaining after 9 h is about 70 mg. By extending the graph, the mass of caffeine after 36 h is about 5 mg.

We can also determine this mass by extending the table.
After 30 h, the mass is 6.25 g.
After 36 h, the mass is 3.125 g.
We get an exact answer using the table.
We assume that the person is not consuming any more caffeine during this time.

4. Use 1-cm grid paper.

a) Draw squares with side lengths from 1 cm to 6 cm.

b) Calculate the area of each square.
Copy and complete this table.

Side length (cm)	Area (cm^2)
0	
1	

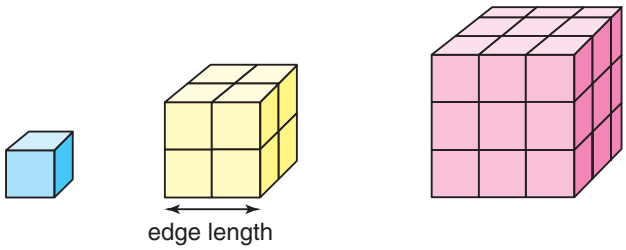
c) Graph the data.
Describe any trends in the graph.

d) Estimate the area of a square with side length 4.5 cm.

e) Estimate the area of a square with side length 7.5 cm.

f) How could you check your answers to parts d and e?

5. **Assessment Focus** The first 3 solids in a pattern are shown.



a) Describe the pattern in the cubes.

b) Sketch the next 2 solids in the pattern.
Copy and complete this table.

Edge length	Number of cubes
1	
2	

c) Describe any trends in the data.

d) Suppose you know the edge length of a solid in this pattern.

i) How can you determine the number of cubes needed to build it?

ii) How is this number related to the volume of the solid?

e) Add rows to your table for the next 2 solids in the pattern.

f) Graph the data.

g) How is the volume of a solid related to its edge length?
Write a rule.

h) What is the edge length of the solid with 512 cubes?

i) How many cubes would be needed for the 10th solid?

j) How many different ways could you answer parts h and i?

6. A new car is purchased for $23 000.
Its value depreciates by 15% each year.
The estimated value of the car over time is shown.

Year	Value of car ($)
0	23 000
1	19 550
2	16 618
3	14 125
4	12 006
5	10 205

a) Predict the shape of the graph. Justify your prediction.
b) Graph the data.
c) Describe the graph.
How does the graph compare to your prediction in part a?
d) Estimate when the value of the car will be about $7500.
How did you do this?

7. In *Investigate*, page 175, you explored the effect of changing the length of a pendulum.

a) Do you think the time it takes for a pendulum to complete 6 swings is related to the mass of the object used?
If your answer is yes, describe how they might be related.
b) Design an experiment you could conduct to test your prediction.
Carry out your experiment. Was your prediction correct?
If you cannot complete the experiment, research to check your prediction.

8. Choose 2 measurements that you think are related.
a) Pose a question about the measurements.
b) Collect data to answer the question. Record the data in a table.
c) Graph the data. Describe any trends in the data.
d) Answer the question you posed.

Use *Fathom* or a graphing calculator, if available.

9. Your teacher will give you a table that shows the mass of caffeine in some foods and drinks.
a) Estimate your total caffeine intake for a typical day.
b) Create a table and graph like those in the *Guided Example*, pages 178-179.
Show the caffeine that remains in your body over time.
Use your answer from part a as the initial value.
c) Repeat parts a and b using data for a friend or family member.

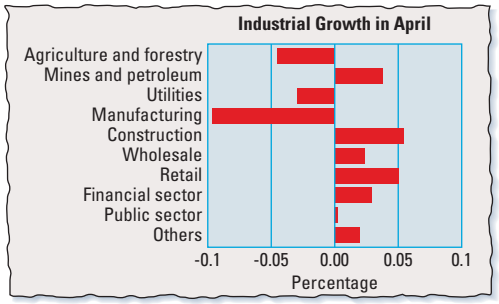
10. **Take It Further** Recall that the formula for the volume of a cone is $V = \frac{1}{3}\pi r^2 h$.
Suppose the height of the cone stays the same, but its radius changes.
Predict if the relationship between the radius and volume is linear or non-linear.
Investigate your prediction. Write what you find out.

In Your Own Words

How can you tell from data that they represent a non-linear relation?
How can you tell from a graph? Include examples in your explanation.

5.6 Interpreting Graphs

Graphs are often used to display information in the media.
These graphs are sometimes misleading or can be misinterpreted.
Knowing how to interpret graphs is an important media literacy skill.



Investigate Representing Motion on a Graphing Calculator

Work in a group of 3.
Your teacher will give you instructions for using the CBR.
You will need: a TI-83 Plus or TI-84 graphing calculator, a CBR motion detector, and a calculator link cable.

You will use the CBR motion detector to investigate changes in your distance from a wall as you move toward and away from the wall.
Connect your graphing calculator to the CBR using a link cable.

- Stand about 3 m away from a wall. Point the CBR at the wall.
  When the CBR starts clicking, walk toward the wall,
  stop for a few seconds, then walk away from the wall.
  Have another group member record a description of your walk.
  Sketch the graph displayed on the calculator.
  What does the vertical axis represent?
  What does the horizontal axis represent?
- Repeat the activity.
  This time, stand close to the wall, walk away from the wall slowly, then quickly. Describe how the graph changes.

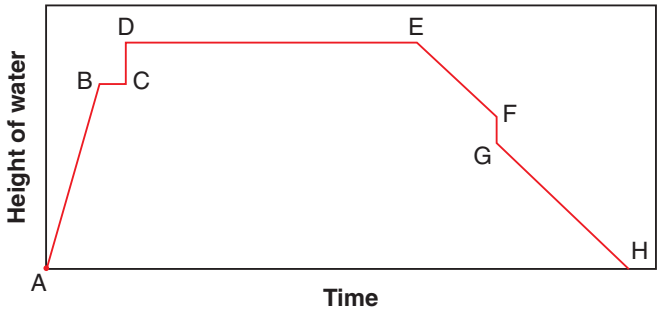
Reflect

Exchange both sketches of your graphs with those of another group.

- Describe the motion represented by each graph.
- Discuss the descriptions the group has written.
  Were the descriptions accurate? Explain.
- What do the shapes of the graphs tell you about the motion?

Connect the Ideas

The graph shows the height of water in a bathtub over time. Key points where the graph changes are labelled.



If we think about the possible reasons for the changes at the key points, we can describe what the graph represents.

At point A, the tub is empty.
A person puts in the plug and turns on the water.
From A to B, the tub fills with water.
At point B, the person turns off the water.
At point C, the person gets into the tub, causing the water level to rise suddenly.
The person sits in the tub from point D to point E.
At point E, he pulls out the plug and the water begins to drain.
At point F, the person gets out of the tub, causing the water level to drop suddenly.
From point G to point H, the water continues to drain from the tub.
At point H, the tub is empty.

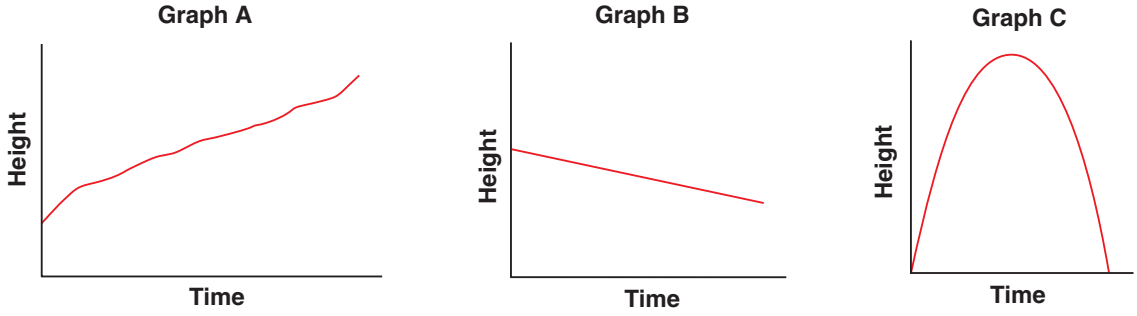
Practice

1. Which graph best represents each situation? Explain your choices.

a) The height of a baseball thrown up into the air measured over several seconds

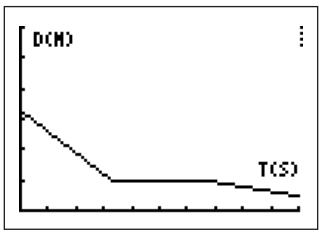
b) The height of a child measured over several years

c) The height of a lit candle measured over several hours

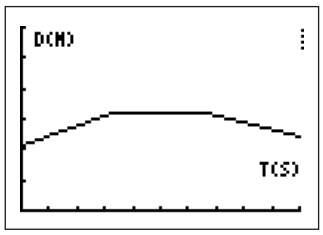


2. Julie and Osa were experimenting with a CBR and made these graphs. Describe their motions.

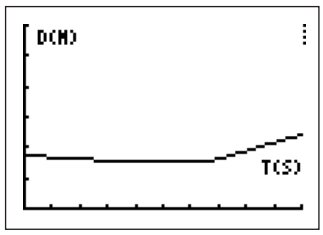
a)



b)



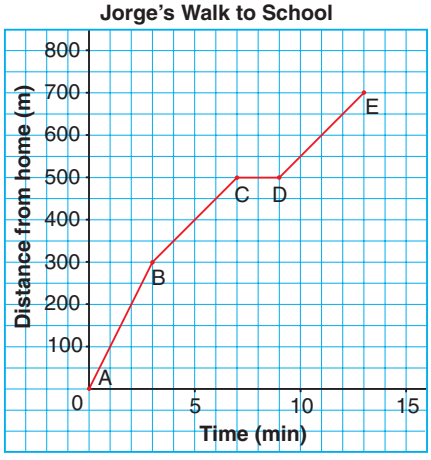
c)



When a graph includes numerical data, we can describe what it shows in more detail.

Example

The graph shows Jorge's distance from home as he walks to school. Describe his walk.



Solution

From A to B, Jorge walks away from home.
The graph goes up to the right.
Jorge is walking for 3 min. He walks 300 m.

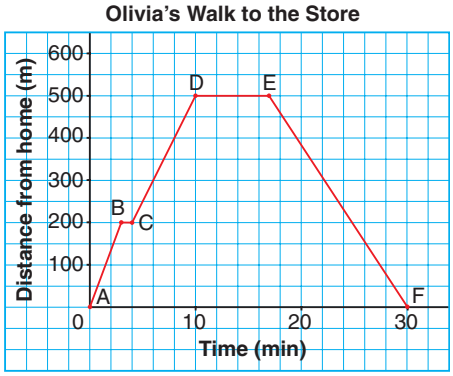
From B to C, Jorge is still walking away from home.
The segment from B to C is less steep than that from A to B.
So, Jorge's average speed has decreased and he is walking slower.
He walks for 4 min and travels 200 m.

From C to D, the segment is horizontal.
This means that time is passing, but Jorge's distance from home stays the same. Jorge is standing still for 2 min.

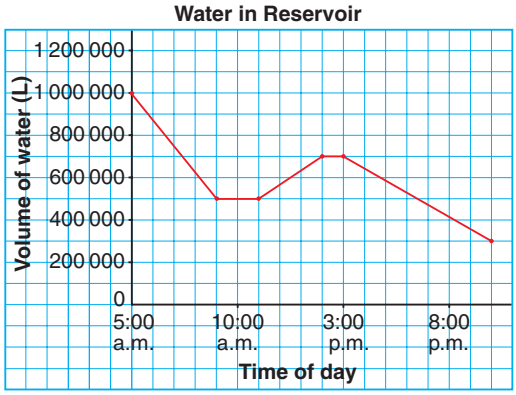
From D to E, Jorge continues his walk to school at about the same average speed as from B to C.
He walks for 4 min and travels 200 m.

It takes Jorge 13 min to walk 700 m to school.

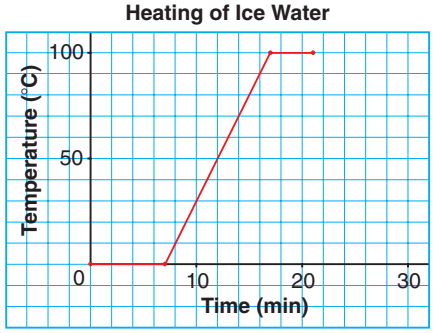
3. The graph shows Olivia's distance from home as she walks to the store and back. Describe her walk.



4. **Assessment Focus** The graph shows how the volume of water in a town reservoir changes during a typical day. Describe how the volume of water changes during the day. Suggest reasons for the changes.



5. **Take It Further** Aaya measured the temperature of a container of ice water as it was heated. She drew this graph. Describe how the temperature changes over time. Suggest reasons for the changes.



In Your Own Words

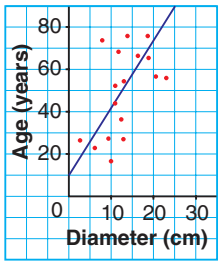
How do the graph title and the axes labels help you interpret data on a graph? Include an example in your explanation.

Chapter Review

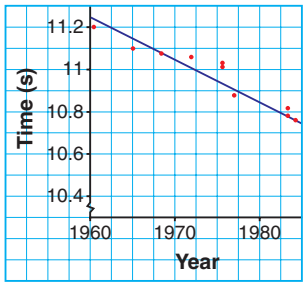
What Do I Need to Know?

Line of Best Fit

Age and Diameter of Northern White Cedar

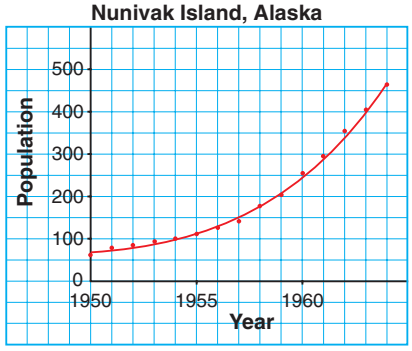


Women's 100-m Track World Records

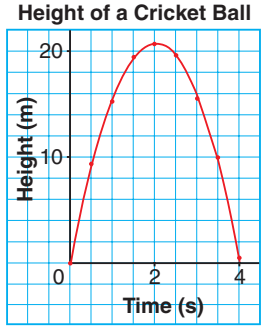


Curve of Best Fit

Muskox Population on Nunivak Island, Alaska



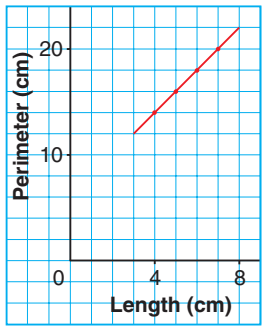
Height of a Cricket Ball



➢ A linear relation has a graph that is a straight line.
For example, the perimeter of a rectangle with width 3 cm:

Length (cm)	Perimeter (cm)
4	14
5	16
6	18
7	20

Perimeter of a Rectangle with Width 3 cm

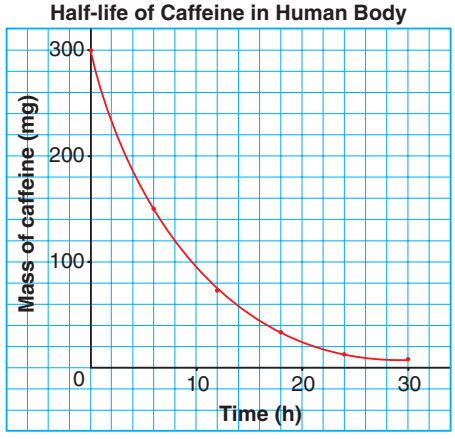


Use a Frayer model to help you understand some of the key words on this page.

➢ A relation that does not have a straight line graph is non-linear.
For example, the half-life of caffeine in the human body:

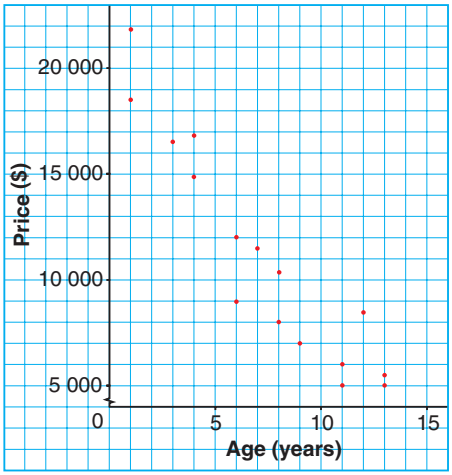
Time (h)	Mass of caffeine (mg)
0	300
6	150
12	75
18	37.5
24	18.75
30	9.375

Half-life of Caffeine in Human Body



What Should I Be Able to Do?

5.1 **1.** a) What does this scatter plot show?



b) How old is the car that costs $16 500?

c) Data are included for two 6-year-old cars.
What are their prices?
Why do you think the prices are different?

d) Describe any trends in the data.
Explain your thinking.

5.2 5.3 **2.** Joseph and Malik measured the growth of a seedling.

Time (days)	1	3	5	7	9
Height (mm)	4	12	22	36	45

a) Graph the data.
Describe any trends.

b) Draw a line or curve of best fit.
Explain how you decided which to draw.

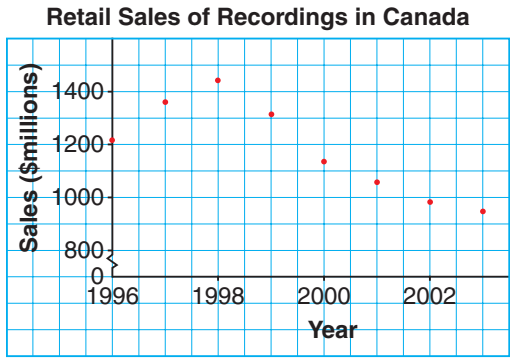
c) i) How tall do you think the seedling was after 4 days?

ii) How tall might it be after 10 days?
How do you know?

d) About how many days might it take for the seedling to reach a height of 60 mm?

e) Can you predict how tall the seedling might be after 1 year?
Explain.

3. The Canadian Recording Industry Association keeps track of sales of recorded music in Canada.
Data for 1996 to 2003 are shown in the graph.



a) Describe any trends in the data.

b) Your teacher will give you a copy of the graph. Draw a curve of best fit.

c) From the graph, about how much did Canadians spend on recorded music in 1998? In 2003?

d) Predict about how much Canadians spent on recorded music in 2004.
What assumptions are you making?

5.2 5.3

4. A mug of hot chocolate was left on a counter to cool. Its temperature was measured every 2 min.

The results are shown in the table.

Time (min)	0	2	4	6	8	10	12	14	16
Temp (°C)	90	86	81	78	74	72	70	68	66

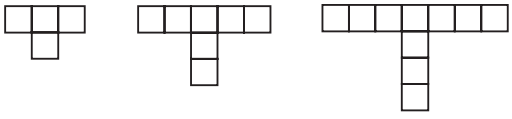
a) Graph the data.
Describe any trends.

b) Draw a line or curve of best fit.
Explain how you decided which to draw.

c) A separate mug of hot chocolate was poured at the same time.
After 1 min some cold milk was added.
How will the graph for this situation differ from the graph you drew in part a?
Sketch the new graph.

5.4

5. The first 3 frames in a pattern are shown.



Frame 1 Frame 2 Frame 3

a) Draw the next 3 frames in the pattern.

b) For each of the 6 frames, record the frame number and the number of squares in a table.

c) Describe the trend.
Do you think the graph will be linear or non-linear?

d) Graph the data. Was your prediction in part c correct?

e) Write a rule for the number of squares in any frame.

f) How many squares will there be in the 8th frame?
How do you know?

g) Which frame will have 28 squares?
How do you know?

5.5

6. Matthew and Tamara are conducting an experiment.
They vary the resistance of a resistor and measure the current through it.
Their results are shown in the table.

Resistance (ohms)	10	20	30	40	50	60
Current (A)	12	6	4	3	2.4	2

a) Describe the trend.

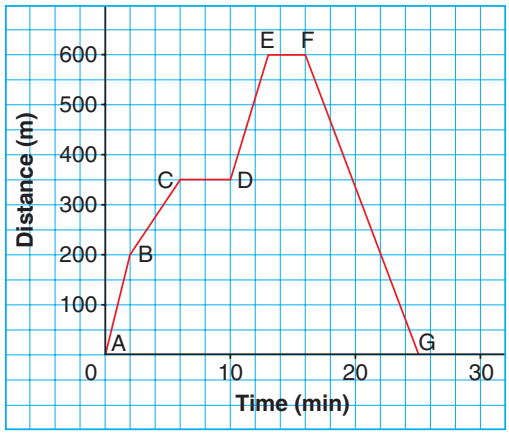
b) Graph the data.

c) What will the current be when the resistance is 15 ohms?

d) What will the resistance be when the current is 10 A?

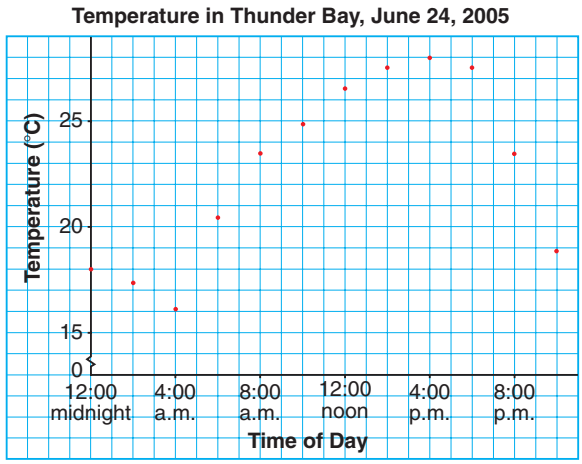
5.6

7. The graph shows Hasieba's distance from home during a walk to the park.
Describe her walk.



Practice Test

Multiple Choice: Use the graph. Choose the correct answer for questions 1 and 2.



1. When was the temperature greatest?

A. 5 p.m. **B.** 2 p.m.
C. 4 p.m. **D.** 11 a.m.

2. Which is the best estimate for the temperature at 7 a.m?

A. 18°C **B.** 20°C
C. 22°C **D.** 25°C

Show your work for questions 3 to 6.

3. Application Water exerts pressure on a scuba diver. The pressure is measured in units called kilopascals (kPa). The table shows the approximate pressure at different depths of sea water.

Depth (m)	Pressure (kPa)
5	150
10	200
15	250
20	300
25	350

a) Graph the data. Draw a curve or line of best fit as appropriate. Explain your choice.
b) At what depth is the pressure 225 kPa? 400 kPa?
c) What is the pressure at the surface of the water?
d) Describe the relationship between depth and pressure.

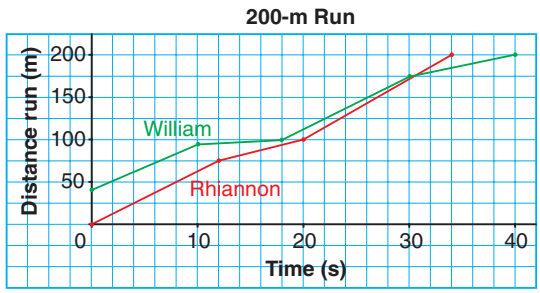
4. Knowledge and Understanding A ball is dropped. The height it reaches after each bounce is measured.
The data are shown in the table.

Bounces	1	2	3	4	5	6
Height (m)	2.1	1.5	1	0.7	0.4	0.3

Which tools could you use to help you?

a) Describe the trend in the data.
b) Graph the data. Draw a curve or line of best fit as appropriate. Explain your choice.
c) From what height do you think the ball was dropped? Explain your answer.
d) What else could you find out from the graph?

5. Thinking William and Rhiannon run 200 m. Describe each person's run.
Who finishes first? Justify your answer.



6. Communication When you look at a graph, how can you tell if it represents a linear relation, a non-linear relation, or neither? Include diagrams in your answer.