What You’ll Learn
To use different proportional reasoning strategies to solve problems involving ratios, rates, and percents

And Why
Situations involving ratios, rates, and percents occur frequently in daily life, such as calculating sales tax or getting the best deal when you shop.

Key Words
- ratio
- equivalent ratios
- proportion
- rate
- unit rate
- percent

Project Link
- Organizing a Banquet
Decoding Word Problems

A word problem is a math question in a real-world context.

Being able to read and understand word problems should help you relate math to life outside the classroom.

Here is a word problem:
Joseph is buying a carpet for his living room.
It is a rectangular room that measures 4 m by 5 m.
The carpet costs $11.99/m².
How much will Joseph spend on carpet?

Many word problems have three parts:

1. The context or set-up
   Joseph is buying a carpet for his living room.

2. The math information
   It is a rectangular room that measures 4 m by 5 m.
The carpet costs $11.99/m².

3. The question
   How much will Joseph spend on carpet?

Some word problems use key words to tell you what to do.
Here are some common key words and phrases:

- Calculate
- Compare
- Estimate
- Explain
- Show your work
- Simplify
- Describe
- Justify
- Solve
- Determine
- List

Work with a partner. Look through the text.
Find an example where each key word is used.
Choose one key word. Explain what it means.
A day-care centre must maintain certain ratios of staff to children. These ratios depend on the age of the child.

For 3- to 5-year-olds, the required ratio of staff to children is $1:10$.

### Investigate

#### Comparing Ratios

Work with a partner.

You will need a ruler or metre stick and 10 copies of the same textbook.

- Measure the height of 1 textbook. Record it in a table.

<table>
<thead>
<tr>
<th>Number of books</th>
<th>Height of pile (cm)</th>
<th>Number of books : height</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Pile the textbooks, one at a time. Record the height of the pile each time you add a book. Stop when your pile reaches 10 textbooks.

- Write the ratio of each number of books to the height.

- What patterns do you see in your table?

### Reflect

Compare your results with those of classmates who used a different textbook.

- How are the patterns the same?
- How are they different?
Recall that a ratio is a comparison of two quantities. For example, suppose the ratio of students to computers in a school is 5:1. This means there are 5 students for every 1 computer.

Equivalent ratios make the same comparison. Suppose a class of 30 students has 6 computers. The ratio of students to computers is 30:6. This ratio is equivalent to 5:1, because it has the same meaning—there are 5 times as many students as computers.

Here are two ways to determine equivalent ratios.

➢ Multiply each term in a ratio by the same number.

\[
8:10 = (3 \times 8) : (3 \times 10) = 24:30
\]

8:10 and 24:30 are equivalent ratios.

➢ Divide each term in a ratio by a common factor.

\[
8:10 = \frac{8}{2} : \frac{10}{2} = 4:5
\]

8:10 and 4:5 are equivalent ratios.

### Practice

1. Determine an equivalent ratio by multiplying.
   
   a) 2:3  
   b) 4:3  
   c) 4:5  
   d) 6:5
2. Determine an equivalent ratio by dividing.
   a) 30 : 50       b) 21 : 7       c) 15 : 12       d) 18 : 36

3. Determine two equivalent ratios for each ratio.
   a) 2 : 5       b) 125 : 500       c) 10 : 1       d) 1500 : 1000

4. The ratio of length to width for this rectangle is 8 : 6.
   a) Use grid paper. Draw a smaller rectangle whose dimensions have the same ratio.
   b) Draw a larger rectangle whose dimensions have the same ratio.
   c) Amir draws a rectangle with length 26 cm and width 18 cm.
      Is the ratio of the dimensions of Amir’s rectangle equivalent to the ratio of the dimensions of your rectangles?
      How do you know?

5. An architect draws a plan for a wheelchair ramp.
   On the plan, the ramp is 2 cm high and 24 cm long.
   What might the dimensions of the actual ramp be?
   How did you use equivalent ratios to find out?

We can use equivalent ratios to compare ratios.

**Example**

June and Megan make grape punch for a party.
June uses grape concentrate and ginger ale in the ratio 3 : 2.
Megan uses grape concentrate and ginger ale in the ratio 8 : 5.
Which punch has the stronger grape taste? Explain.

**Solution**

The recipe that uses more concentrate for the same volume of ginger ale has the stronger grape taste.

The volume of ginger ale is the second term of each ratio.
So, write 3 : 2 and 8 : 5 with the same second term.
The least common multiple of 2 and 5 is 10.
So, write both 3 : 2 and 8 : 5 with the second term 10.

- **June’s punch**
  
  \[3 : 2 = (3 \times 5) : (2 \times 5) = 15 : 10\]

- **Megan’s punch**
  
  \[8 : 5 = (8 \times 2) : (5 \times 2) = 16 : 10\]

The volume of concentrate is the first term of each ratio.
Megan’s punch uses more concentrate for the same volume of ginger ale.
It has the stronger grape taste.
6. Refer to the Guided Example, page 113.
   a) Write equivalent ratios for $3:2$ and $8:5$ with the same first term.
   b) How do the ratios in part a help you determine which punch has the stronger grape taste?

7. Compare the strawberry punch in each pair of pitchers.
   Which has the stronger strawberry taste? Explain.
   a) 
   ![Pitcher A](image1.png) ![Pitcher B](image2.png)
   b) 
   ![Pitcher A](image3.png) ![Pitcher B](image4.png)

8. **Assessment Focus** Beakers of blue and clear liquids are combined.
   a) Predict which set you think will produce the bluer liquid. Justify your answer.

   ![Set A](image5.png) ![Set B](image6.png)

   b) Which set does produce the bluer liquid? Explain.
   c) Sketch a different set of beakers that would produce the same shade of blue as set A.

9. A loft measures 10 m by 16 m.
   A scale drawing of the loft is to fit on a 21-cm by 28-cm sheet of paper.
   What could the dimensions of the scale drawing be? How did you find out?

10. **Take It Further** A CD jewel case measures 12.5 cm by 14.0 cm by 1.0 cm.
    A model of the case measures 5.0 cm by 5.6 cm by 0.5 cm.
    One dimension on the model is incorrect.
    a) Which dimension is incorrect?
    b) What should the dimension be?

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**In Your Own Words**

What are the equivalent ratios?
Explain two ways to determine a ratio that is equivalent to a given ratio.
Include examples in your explanation.
A nurse uses equivalent ratios to calculate drug dosages.
A drug comes in 150-mg tablets.
The dosage ordered is 375 mg.
The nurse needs to calculate how many tablets are required.

Investigate
Using Proportional Reasoning to Solve a Problem

Use counters.
A hospital gift shop sells 4 magazines for every 3 books.
➢ Suppose the shop sold 18 books. How many magazines did it sell?
➢ Suppose the shop sold 12 magazines. How many books did it sell?
➢ One Friday, the shop sold 35 books and magazines altogether. How many of each kind did it sell?

Record your work.

Reflect
Compare your recording strategies with those of a classmate.
➢ How could you solve the problems without using counters?
➢ How can you use equivalent ratios to solve the problems?
CHAPTER 4: Proportional Reasoning

When the body mass doubles, the dose doubles.

When the body mass triples, the dose triples.

We say that the drug dose is proportional to the body mass. 5:60, 10:120, and 15:180 are equivalent ratios.

We can determine the drug dose for a body mass of 35 kg. Let $d$ milligrams represent this dose.

We need a ratio equivalent to $5:60$, with the first term 35.

That is, $5:60 = 35:d$

Here are two ways to determine the value of $d$.

**Look for a multiplication relationship between ratios**

$\times 7$

$5:60 = 35:d$

Think: What do we multiply 5 by to get 35? Multiply 60 by the same number.

So, $60 \times 7 = d$

That is, $d = 420$

**Look for a multiplication relationship within ratios**

$\times 12$

$5:60 = 35:d$

Think: What do we multiply 5 by to get 60? Multiply 35 by the same number.

So, $35 \times 12 = d$

That is, $d = 420$

A dose of 420 mg is needed for a body mass of 35 kg.

Hamadi is a pediatric nurse.

He uses a table like the one below to give the correct dose of a pain reliever.

<table>
<thead>
<tr>
<th>Approximate body mass (kg)</th>
<th>5</th>
<th>10</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dose (mg)</td>
<td>60</td>
<td>120</td>
<td>180</td>
</tr>
</tbody>
</table>

In proportional situations, the quantities involved are related by multiplication or division.

The statement $5:60 = 35:d$ is a proportion. A proportion is a statement that two ratios are equal.
1. Describe two ways the numbers in each proportion are related.
   a) 5 : 20 = 125 : 500  
   b) 10 : 1 = 120 : 12  
   c) 75 : 25 = 300 : 100  
   d) 1 : 3 = 16 : 48

2. Multiply between ratios to determine each value of $n$.
   a) $2 : 5 = 8 : n$  
   b) $2 : n = 6 : 9$  
   c) $n : 5 = 12 : 20$  
   d) $8 : n = 4 : 15$

3. Multiply within ratios to determine each value of $z$.
   a) $4 : 8 = 3 : z$  
   b) $5 : z = 6 : 18$  
   c) $z : 14 = 10 : 20$  
   d) $3 : 21 = z : 56$

4. A portable music player with 4 GB of memory stores about 1000 songs.
   A music player with 60 GB of memory stores about 15 000 songs.
   Is the number of songs proportional to the amount of memory?
   Explain your reasoning.

5. To make green paint, 3 parts yellow paint are mixed with 2 parts blue paint.
   Janis has 12 L of blue paint. How much yellow paint does she need? Explain.

6. Ali earned $80 working 10 h.
   How long would it take him to earn $200? Explain how you found your answer.

7. A recipe that serves 4 people uses 3 potatoes.
   How many potatoes are needed to serve 20 people?

   Sometimes, it is helpful to simplify one of the ratios in a proportion.

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**Example**

Determine the value of $c$.

$c : 20 = 18 : 15$

**Solution**

$c : 20 = 18 : 15$

We cannot immediately identify how the terms are related.

$c : 20 = 18 : 15$

Use mental math to determine an equivalent ratio for 18 : 15.

Divide each term by 3.

$18 : 15 = \frac{18}{3} : \frac{15}{3}$

$= 6 : 5$

So, $c : 20 = 6 : 5$

$\div 4$

Since $20 \div 4 = 5$, then $c \div 4 = 6$

So, $c = 24$
8. Determine the value of each variable.
   a) $4:10 = 18:c$   b) $125:25 = n:6$   c) $6:y = 9:12$   d) $60:z = 24:6$

9. **Assessment Focus** In 1996, the Royal Canadian Mint issued the new $2 coin.
   a) A poster advertising the new coin showed a large photograph of the toonie.
      The diameter of the inner core on the poster was 51 cm. What was the outer diameter of the coin on the poster? Show your work.
   b) Did you solve the problem using a proportion? How could you solve it without using a proportion?

10. When a robin flies, it beats its wings about 23 times in 10 s. How many times will it beat its wings in 2 min? Explain your thinking.

11. A gear ratio is the ratio of the numbers of teeth in two connected gears. The gear ratio of two gears is $3:2$.
    a) The larger gear has 126 teeth. How many teeth does the smaller gear have? How did you solve the problem?
    b) Suppose the smaller gear has 126 teeth. How many teeth would the larger gear have? Explain. Show your work.

12. **Take It Further** There are 900 students enrolled in Mount Forest Secondary School. The ratio of girls to boys is $5:4$.
    a) How many boys and how many girls go to Mount Forest SS? Explain how you found your answer.
    b) The average class size is 27 students. Suppose this class is representative of all the students in the school. How many students in this class are girls? How many are boys?

**In Your Own Words**
Use one of the questions in this section. Explain how you can use a proportion to answer the question.
Identifying Scales

1. Open *The Geometer’s Sketchpad*. 
   Open the sketch `Investigating Scale Drawings.gsp`. 
   The sketch has scale drawings of some rooms in a house. 
   The actual dimensions of each room are shown.

   - **Kitchen**  
     7 m by 4 m
   - **Bedroom**  
     3 m by 5 m
   - **Breakfast Area**  
     4 m by 3 m
   - **Living Room**  
     4 m by 7 m

   Use the instructions your teacher gives you.

2. Measure the lengths of the sides of each room.

3. Compare the actual measures on the diagram to the lengths you measured.
   a) Which rooms are drawn to scale? How do you know?
   b) What is the scale?

4. a) Which rooms are not drawn to scale? Explain.
   b) Suppose these rooms were drawn using the scale in question 3.
      What would the dimensions be?
Creating a Scale Drawing

1. Open a new sketch.

2. You will create a scale drawing of a soccer field. The actual dimensions of the field are:
   - Touchline – 120 m
   - Goal line – 90 m
   - Penalty area – 16.5 m by 40 m
   - Centre circle – radius of 9.15 m

3. Choose the scale you will use to create your drawing.

4. Follow the instructions to create the scale drawing using the scale you chose. How do you determine the length and width of your scale drawing?

5. How would the drawing change if you used a smaller scale? A larger scale? Construct new scale drawings to check your predictions. Did you use equivalent ratios? Explain.

6. Print your scale drawings. Trade drawings with a classmate. Determine the scale for each of your classmate’s drawings.

7. Construct a scale drawing of an object of your choice. Print the drawing and explain how you constructed it. Include the actual dimensions of the object and the scale.
In many places in Ontario, home-owners use firewood to heat their homes. The firewood is split and stacked into units called *cords*. One cord of wood measures about 1.20 m wide by 2.40 m long by 1.20 m high.

### Investigate Comparing Unit Rates

You will investigate two methods in which firewood is split.

- Northern Lights Firewood uses a machine called a firewood processor. It produces 32 cords in 8 h.
- Duhaime’s Timber Products uses 6 people with chain saws and log splitters. They produce 21 cords in 6 h.

Which company produces firewood faster?

### Reflect

- What strategy did you use to determine the faster method?
- Compare your strategy for solving the problem with those of other students. If you used different strategies, explain your strategy to other students.
To determine which company has the greater operating costs, we determine the operating costs per hour.

Northern Lights Firewood’s operating costs are $620 for 8 h. To determine the hourly operating costs, divide 620 by 8.

\[
\frac{620}{8} = 77.5
\]

The operating costs are $77.50 per hour.

Duhaime’s Timber Products’ operating costs are $585 for 6 h. To determine the hourly operating costs, divide 585 by 6.

\[
\frac{585}{6} = 97.5
\]

The operating costs are $97.50 per hour.

So, Duhaime’s Timber Products has the greater operating costs.

A rate is a comparison of two quantities with different units.

Each hourly rate is called a unit rate, because it tells the cost for 1 h, or one unit of time.

Unit rates can be written in different ways:

- Using words: one hundred kilometres per hour
- Using numbers, symbols, and words: 100 km per hour
- Using numbers and symbols: 100 km/h
Practice

1. Determine each unit rate.
   a) 50 goals scored in 25 games
   b) $400 earned in 40 h
   c) $6.00 for 12 oranges
   d) 770 km travelled in 7 h

2. Determine each unit rate.
   a) $5.00 for 10 CDs
   b) $2.37 for 3 kg of apples
   c) 9 kg lost in 6 weeks
   d) 22 km hiked in 5 h

3. Twelve hundred litres of water were pumped out of a flooded basement in 8 h. What was the unit rate of outflow?

4. Mr. and Mrs. Dell recently finished some renovations to their house. Twenty square metres of carpeting cost $875. Seventeen square metres of hardwood flooring cost $750. Which flooring is more expensive per square metre? Explain.

5. Alec wants to join one of three online music clubs.
   eTunes
   $21.00 for 20 songs
   Tunezilla
   $41.65 for 35 songs
   Monstersongs.com
   $54.50 for 50 songs

   Which music club is more economical? Explain.

6. A 12-pack of pop costs $2.99. The same pop costs $0.75 per can from a vending machine. Is this a fair price? Explain.

We can use unit rates to determine the best buy.

Example

Cereal comes in 3 sizes.

<table>
<thead>
<tr>
<th>Size</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>500 g</td>
<td>$2.99</td>
</tr>
<tr>
<td>650 g</td>
<td>$3.69</td>
</tr>
<tr>
<td>800 g</td>
<td>$4.29</td>
</tr>
</tbody>
</table>

a) Calculate the unit price for each box.
b) Which box is the best buy? Explain.
Solution  
a) Since 1 g is very small, we calculate the cost of 100 g.  
This is the unit price.  
Box A has mass 500 g and costs $2.99.  
500 g is $5 \times 100$ g; so, the cost of 100 g of Box A is: $\frac{2.99}{5} = 0.598$  
Box B has mass 650 g and costs $3.69.  
650 g is $6.5 \times 100$ g; so, the cost of 100 g of Box B is: $\frac{3.69}{6.5} = 0.568$  
Box C has mass 800 g and costs $4.29.  
800 g is $8 \times 100$ g; so, the cost of 100 g of Box C is: $\frac{4.29}{8} = 0.536$

b) Box C has the lowest unit price. It is the best buy.

7. Which is the best buy? Show your work.

![Image of juice options]

8. **Assessment Focus** Sue wants to buy tea.  
The brand that she buys comes in 2 sizes.  
a) Use unit rates.  
Which box of tea bags is the better buy?  
b) How could Sue determine the better buy without using unit rates? Explain.  
c) Why might Sue not want to purchase the better buy?

9. **Take It Further** A 400-g package of Grandma's Cookies has 18 cookies and costs $2.49.  
A 700-g package of Dee's Delights has 12 cookies and costs $3.99.  
a) Which unit would you use to determine the better buy? Explain why you chose that unit.  
b) Determine the unit price of each brand of cookie.  
c) Which brand is the better buy? Explain.

### In Your Own Words

What is a unit rate?  
How can you use unit rates to determine the better buy?  
Use examples in your explanation.
Grid Paper Pool

Materials
Grid Paper Pool Master
1-cm grid paper

Each “pool table” is a rectangle drawn on a grid. There is a pocket at each corner. The figure at the right is a 3 by 2 table.

Here are the rules of the puzzle.
➢ An imaginary ball is hit from corner A. It travels across the grid, at 45° to AB.
➢ The ball bounces off each side it hits at a 45° angle.
➢ The ball continues to travel until it reaches a pocket.

Your teacher will give you copies of different “pool tables.”
➢ For each pool table:
  – Draw the path the ball takes.
  – Copy and complete this table.
  – Write the pocket letter in the third column.

<table>
<thead>
<tr>
<th>Pool table dimensions</th>
<th>Number of hits</th>
<th>Ball reaches pocket:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

➢ What patterns do you see in the results?
➢ Another table measures 16 by 24. Predict the number of hits before the ball stops. Predict the pocket in which the ball stops. How did you make your predictions?
➢ Determine the dimensions of 2 different pool tables where the ball stops in pocket D after 7 hits.
4.1 1. Which ratios in each pair are equivalent? How do you know?
   a) 12 : 15 and 48 : 60
   b) 3 : 2 and 25 : 16
   c) 3 : 4 and 15 : 18
   d) 625 : 125 and 180 : 36

2. Lemonade is made with water and concentrate.
   a) Which pitcher has the stronger lemonade? Explain how you know.

   b) Describe the contents of a different pitcher that has the same strength lemonade as Pitcher B.

4.2 3. A 7500-kg African elephant eats about 200 kg per day. A 5000-kg African elephant eats about 150 kg per day. Is the amount of food an elephant eats proportional to its mass? Explain your reasoning.

4. A recipe for salad dressing uses 60 mL of vinegar for 240 mL of oil. Emil has only 40 mL of vinegar. How much oil should he use?

6. In 2002, 3 out of 10 doctors in Canada were women.
   a) What is the ratio of female doctors to male doctors?
   b) There are 72 female doctors in a city. How many male doctors are there?
   c) There are 119 male doctors in another city. How many female doctors are there?
   d) What assumptions did you make in parts b and c?

4.3 7. Calculate each unit rate.
   a) Driving 45 km on 3 L of gas
   b) Typing 120 words in 4 min
   c) Earning $148 for an 8-h shift
   d) Driving 525 km in 6 h


9. a) Determine the price for 100 g of each box of cereal.

   b) Which box is the better buy? Explain.
Cary wants to estimate the distance between two places. He uses a map scale and proportional reasoning.

Many different types of problems can be solved using proportional reasoning.

**Investigate Using Different Strategies to Solve a Problem**

➢ Use Pattern Blocks to create this design.

![Pattern Blocks Design](image)

Each block in the design has a value that is proportional to its area.
The value of the yellow hexagon is $3.
What is the value of the entire design?
How do you know?

➢ Create a design that costs between $5 and $6.
Write the value of your design.
Record your design.
Explain how you determined the value.

**Reflect**

Compare strategies with your classmates.

➢ How could you use equivalent ratios to solve the problems?

➢ How could you use unit rates to solve the problems?
Kirsten’s neighbour used 120 patio stones to cover 8 m². Kirsten wants to build a patio using the same type of stone.

Kirsten wants to pave 18 m² of her backyard. We can use unit rates to determine the number of patio stones Kirsten needs.

An area of 8 m² requires 120 patio stones. So, an area of 1 m² requires: \[
\frac{120}{8} = 15 \text{ stones}
\]

Then, an area of 18 m² requires: \[
18 \times 15 = 270 \text{ stones}
\]

Kirsten needs 270 stones to pave her backyard.

Kirsten discovers that the patio stones are on sale. She can afford to buy 330 stones. Kirsten decides to build a larger patio.

We can use a similar method to determine the greatest area Kirsten can cover with 330 stones.

\[
\begin{array}{c|c|c|c|c}
\div 120 & 120 & 8 & \div 120 \\
\times 330 & 330 & \frac{8}{120} & \times 330 \\
\end{array}
\]

\[
330 \times \frac{8}{120} = 22 \text{ m}^2
\]

The greatest area Kirsten can cover with 330 stones is 22 m².
Practice

1. Sean buys 8 DVDs for $120.
   Each DVD costs the same amount.
   a) How much does 1 DVD cost?
   b) How much do 13 DVDs cost?

2. Ioana works for 4 days and earns $224. At this rate:
   a) How much does she earn in 1 day?
   b) How much does she earn in 11 days?

3. Mika worked 6 shifts and earned $720.
   She earns the same amount for each shift.
   How much money would she earn in 4 shifts?

   a) How many cases could he buy for $91? Explain.
   b) How could you have solved the problem a different way?

5. A car travels 96 km on 8 L of gasoline.
   a) How far will the car travel on 25 L of gasoline?
   b) How much gasoline is needed to travel 120 km?
      Show your work.

6. A machine can bind 1000 books in 8 min.
   a) How many books can the machine bind in 1 h?
   b) How long will it take to bind 7250 books? Explain.

We use proportional reasoning when we use a map to determine distances.

Example

The driving distance from Sault Ste. Marie to North Bay is about 425 km.
On a map, this distance is 8.5 cm.
On the same map, the distance from North Bay to Windsor is 13.5 cm.
What is the actual distance between these cities?

Solution

8.5 cm represents 425 km.
So, 1 cm represents: \( \frac{425}{8.5} \) km = 50 km
Then, 13.5 cm represents: \( 13.5 \times 50 \) km = 675 km
The actual distance from North Bay to Windsor is about 675 km.
7. Madurodam is a miniature city in the Netherlands. Every object in the city is built to the same scale. A lamppost in Madurodam is 12 cm tall. It is modelled after an actual lamppost that is 3 m tall.
   a) A bridge that crosses the canal in Madurodam is 30 cm long. How long is the actual bridge that was used as the model?
   b) A clock tower is to be built in Madurodam. It is modelled after a clock tower that is 25 m tall. How tall should the miniature tower be?
   c) Explain how you solved each problem.

8. Chloë challenged her father to a basketball game. Each person gets a different number of points for scoring a basket.
   a) Chloë scored 12 baskets and has 36 points. How many points would she get if she scored 15 baskets?
   b) Her father scored 16 baskets and has 32 points. How many points would he get if he scored 19 baskets?
   c) Their total score at the end of the game is 95 points. How many baskets did each person score?

9. Assessment Focus Imran is visiting his grandmother in the United States. He can exchange $20 Canadian for $17 US.
   a) How many US dollars would Imran get for $225 Canadian?
   b) When Imran returns, he has $10 US. How much is this in Canadian dollars? What assumptions did you make?

10. Take It Further A recipe for pancakes calls for 2 cups flour mixed with $1 \frac{1}{2}$ cups buttermilk. Liisa mixes 5 cups flour with $2 \frac{1}{2}$ cups buttermilk. Will Liisa’s pancake mix be too dry, too watery, or just right? How do you know?

In Your Own Words
You have learned several strategies to solve proportional reasoning problems. Choose one strategy. Explain when you would use it. Include an example of a problem.
Hockey sticks vary in length. Taller players use longer sticks. The National Hockey League only allows hockey sticks that are shorter than 1.60 m.

Investigate

Solving an Equation to Solve a Problem

In 2005, Zdeno Chara was a defenceman for the Ottawa Senators. He was the tallest player in the league. The NHL made an exception to the stick length rule for Chara because he was so tall. At 2.06 m, Chara used a stick that was 1.80 m long.

➢ The shortest player in the NHL has a height of 1.70 m. Suppose the ratio of the length of his stick to his height is the same as it is for Chara. What is the length of the stick?

➢ The Ottawa Senators sometimes give out souvenir miniature sticks at their games. How short would a player have to be to use a stick 37 cm long? What assumptions do you make?

Reflect

Compare strategies for solving these problems with your classmates.

➢ Did you write a proportion in each case? Explain.

➢ What other strategies could you have used to solve these problems?

➢ How could you check your answers?
The Toronto Maple Leafs scored 17 power-play goals in the first 8 games of the 2005–2006 season. Suppose the team continued to score power-play goals at the same rate. We can estimate how many of these goals it might score in the 82-game season.

Let \( n \) represent the number of power-play goals scored in 82 games.

Here are two ways to determine the value of \( n \).

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**Solve a proportion using within ratios**

17 goals ... in ... 8 games

\[
\frac{17}{8} = \frac{n}{82}
\]

Write each ratio in fraction form.

To isolate \( n \), multiply each side of the equation by 82.

\[
82 \times \frac{17}{8} = n \times 82
\]

\[
\frac{82 \times 17}{8} = \frac{82n}{82}
\]

\[
\frac{1394}{8} = n
\]

\[
\boxed{n = 174.25}
\]

---

**Solve a proportion using between ratios**

17 goals ... in ... 8 games

\[
\frac{17}{8} = \frac{n}{82}
\]

Write each ratio in fraction form.

To isolate \( n \), multiply each side of the equation by 17.

\[
17 \times \frac{n}{17} = 82 \times \frac{8}{17}
\]

\[
\frac{17n}{17} = \frac{82 \times 17}{8}
\]

\[
\boxed{n = \frac{1394}{8}}
\]

If the Leafs continued at the same rate as for the first 8 games, they would have scored about 174 power-play goals in the 2005–2006 season.

The ratio of power-play goals to games is 17:8. This is approximately 16:8, or 2:1. So, the number of power-play goals is about double the number of games. Then, in 82 games, the number of power-play goals would be about: \( 82 \times 2 = 164 \) This is close to the calculated number of 174.
1. Solve for \(n\).
   a) \(\frac{n}{6} = \frac{3}{2}\)
   b) \(\frac{n}{4} = \frac{6}{8}\)
   c) \(\frac{n}{12} = \frac{5}{4}\)
   d) \(\frac{n}{25} = \frac{8}{5}\)
   e) \(\frac{n}{16} = \frac{5}{8}\)

2. Solve for each variable.
   a) \(c : 9 = 21 : 7\)
   b) \(m : 840 = 1 : 120\)
   c) \(y : 63 = 5 : 3\)
   d) \(a : 225 = 7 : 9\)

The length of an object’s shadow is proportional to the object’s height.

**Example**

A person is 1.85 m tall.
At a certain time of day, her shadow is 0.74 m long.
At the same time of day, the shadow of a tree is 8.40 m long.
How tall is the tree?

![Diagram of person and tree with shadows]

**Solution**

Let \(n\) metres represent the height of the tree.
The ratio of the person’s height to the length of her shadow is \(1.85 : 0.74\).
The ratio of the tree’s height to the length of its shadow is \(n : 8.40\).
Write a proportion.
\[\frac{1.85}{0.74} = \frac{n}{8.40}\]
Write each ratio in fraction form.
\[\frac{1.85}{0.74} = \frac{n}{8.40}\]
To isolate \(n\), multiply each side of the equation by 8.40.
\[8.40 \times \frac{1.85}{0.74} = \frac{n}{8.40} \times 8.40\]
\[\frac{8.40 \times 1.85}{0.74} = \frac{8.40n}{8.40}\]
Simplify the equation.
\[\frac{15.54}{0.74} = n\]
\[n = 21\]
The tree is 21 m tall.
3. Each shadow is measured at the same time as the shadows in the Guided Example on page 133.
   a) A toddler casts a shadow that is 0.33 m long. How tall is the toddler?
   b) A flagpole casts a shadow that is 2.96 m long. How tall is the pole?

4. At an automotive repair shop, 3.5 h of labour costs $311.50. What is the labour charge for a 5-h job? How could you check your answer?

5. **Assessment Focus**
   a) In 2005, the band U2 sold out two concerts in Dublin, Ireland. Forty-eight thousand tickets were sold in the first 15 min. Assume the tickets sold at a constant rate. How many tickets were sold in 50 min?
   b) Seventy-five thousand tickets were sold at the Stade de France in Paris, France. Suppose the tickets sold at the same rate as in Dublin. How long did it take to sell the 75 000 tickets?
   c) Explain the strategies you used to solve the problems.

6. **Take It Further** Leonardo Da Vinci believed in ideal proportions of the body.
   In his paintings, the ratio of a person’s height to the floor-to-hips height is $7 : 4$.
   a) A person has a floor-to-hips height of 1.16 m. How tall is the person?
   b) Elisha is 167 cm tall. What is her floor-to-hips height?
   c) Determine whether the ratio of your height to floor-to-hips height is the same as Da Vinci’s ratio. Compare your findings with those of your classmates.

---

**In Your Own Words**

Use one of the questions in this section. If you solved the problem using an equation, find a different way to solve it. If you did not use an equation to solve the problem, solve it now by using an equation.
Investigate

Using Percent to Compare Raises

Work with a partner.

Elena, Shane, and Gillian worked last summer at different retail stores. This summer, all of them received raises.

Elena’s wages increased from $360 to $425 per week. Shane’s wages increased from $275 to $340 per week. Gillian’s wages increased from $400 to $470 per week.

➢ Who received the greatest raise?
➢ Compare each person’s raise with the previous year’s wages. Who received the greatest percent increase?

Explain how you know.

Reflect

Compare strategies with your classmates for solving these problems.

➢ You represented each pay raise in two ways. Which way do you think better represents the pay raise? Explain.
A **percent** is a ratio that compares a number to 100; for example, 30 : 100.

A percent can also be written with a symbol (%), as a fraction, or as a decimal.

For example, $30\% = \frac{30}{100} = 0.30$

An MP3 player that regularly sells for $180 is on sale for 25% off.

The sale price is $100% - 25% = 75\%$ of the original price, or 75\% of $180.

So, the sale price is in the same ratio to 180 as 75 is to 100.

Let $b$ dollars represent the sale price.

$b : 180 = 75 : 100$

Write this proportion in fraction form.

$$\frac{b}{180} = \frac{75}{100} = 0.75$$

To isolate $b$, multiply each side of the equation by 180.

$$\frac{b}{180} \times 180 = 0.75 \times 180$$

$$b = 135$$

The sale price is $135.

We have to pay 14\% in taxes on the MP3 player. So, the total price is $100\% + 14\% = 114\%$ of the sale price, or 114\% of $135$.

Let $t$ dollars represent the total price.

$$\frac{t}{135} = \frac{114}{100} = 1.14$$

To isolate $t$, multiply each side of the equation by 135.

$$\frac{t}{135} \times 135 = 1.14 \times 135$$

$$t = 153.9$$

The total price of the MP3 player is $153.90.
1. Write each fraction as a decimal.
   a) \( \frac{7}{100} \)  \hspace{1cm} b) \( \frac{15}{100} \)  \hspace{1cm} c) \( \frac{35}{100} \)  \hspace{1cm} d) \( \frac{80}{100} \)  \hspace{1cm} e) \( \frac{120}{100} \)

2. Determine each value.
   a) 10% of $365  \hspace{1cm} b) 25\% \text{ of } 50 \text{ kg}  \hspace{1cm} c) 50\% \text{ of } 28 \text{ m}  \hspace{1cm} d) 125\% \text{ of } 120 \text{ g}

3. A jacket was regularly priced at $159.99.
   It was marked down by 30%. What was the sale price of the jacket?

4. During January, 50,000 new vehicles were sold in Ontario.
   About 20% of these were leased. How many vehicles were leased?

   a) How much is the sales tax on the game?
   b) What is the price including taxes?

   They are on sale at 45% off. What is the total cost, including taxes?
   Think of a different way to solve this problem.

7. There were 288 spectators at the football game.
   75\% were cheering for the home team.
   a) How many spectators were cheering for the home team? Explain.
   b) 40\% of the spectators were students.
      How many spectators were adults? How do you know?

We use percents when we calculate simple interest.

**Example**

Emma borrows $1200 for 6 months.
The annual interest rate is 6%.
How much simple interest does Emma pay?

**Solution**  
*Method 1: Use ratios*

For one year, the ratio of simple interest to the loan is equal to the ratio of the interest rate to 100%.
Let \( x \) dollars represent the simple interest.
Then, \( \frac{x}{1200} = \frac{6}{100} \)

\[ 1200 \times \frac{x}{1200} = \frac{6}{100} \times 1200 \]

\[ x = \frac{7200}{100} \]

\[ x = 72 \]
The interest for 1 year is $72, so the interest for 6 months is: \( \frac{72}{2} = 36 \)

**Method 2:** Use algebra

a) Use the formula: \( I = Prt \)

\( I \) is the simple interest in dollars.

The principal, \( P \), is $1200.

The annual interest rate, \( r \), is 6%, or 0.06.

Since time, \( t \), is measured in years, write 6 months as a fraction of a year: \( \frac{6}{12} \)

Substitute: \( P = 1200, r = 0.06, \) and \( t = \frac{6}{12} \)

So, \( I = 1200 \times 0.06 \times \frac{6}{12} \)

\( = 36 \)

Emma pays $36 simple interest.

8. Connor borrows $5000 for 9 months. The annual interest rate is 8%. How much simple interest does Connor pay?

9. John put $500 in a savings account for 8 months. The annual interest rate is 2%.
   a) How much simple interest does the money earn?
   b) How much money is in the account after 8 months?

10. **Assessment Focus** A credit card company charges 24% per year on outstanding balances.
    a) How much interest would be charged on an outstanding balance of $900 for 90 days?
    b) How much is owed at the end of 90 days?
    Show your work.

11. **Take It Further** Marie borrowed $3500 for 6 months.
    She paid $140 simple interest.
    What was the annual interest rate?
    How could you check your answer?

**In Your Own Words**

What do you find most challenging when you solve problems involving percent?
Use a question from this section to explain.
How might you overcome this difficulty?
What Do I Need to Know?

A ratio is a comparison of two quantities.
Two ratios are equivalent when they can be reduced to the same ratio.
For example, both 12 : 16 and 9 : 12 reduce to 3 : 4,
so they are equivalent ratios.

![Image showing equivalent ratios]

A proportion is a statement that two ratios are equal.
For example, 2 : 3 = 10 : 15
or, \( \frac{2}{3} = \frac{10}{15} \)

To solve a proportion means to determine the value of an unknown term in a proportion.
For example, to solve 10 : 15 = n : 3, determine the value of n that makes the ratios equal.

A rate is a ratio of two terms with different units.
A unit rate is a rate where the second term is 1 unit.
For example, 50 km : 1 h is written as 50 km/h.

A percent is a ratio with second term 100.
For example, 25 : 100 can be written:
in fraction form as \( \frac{25}{100} \),
in decimal form as 0.25, and
as a percent, 25%

![Image showing percentage scale]
What Should I Be Able to Do?

1. Write 2 equivalent ratios for each ratio.
   a) 3 : 5  
   b) 36 : 42  
   c) 15 : 10  
   d) 225 : 35

2. Elise made up a game with this spinner.

   ![Spinner Diagram]

   If the pointer lands on red, the player loses a turn.
   Elise made a larger spinner. It had 20 sectors, 12 of which are red.
   a) Are the spinners equivalent?
   b) If your answer to part a is yes, justify your answer.
      If your answer is no, describe a spinner that would be equivalent to the first spinner.

3. Orange juice is mixed from concentrate and water.

   ![Orange Juice Diagram]

   Which mix is stronger?
   Justify your answer.

4. Determine the value of each variable.
   a) \(3 : 4 = 9 : n\)
   b) \(a : 35 = 4 : 20\)
   c) \(10 : 20 = 21 : e\)
   d) \(20 : m = 15 : 18\)

5. A recipe for salad dressing calls for 4 parts oil to 1 part vinegar. Allie used 60 mL of vinegar. What volume of oil does she need?

6. A recipe that makes 5 dozen cookies calls for 4 eggs and 2 cups of flour.
   a) Wolfgang has only 3 eggs.
      How much flour should he use?
   b) How many cookies will he make?

7. Determine each unit rate.
   a) 240 km driven in 5 h
   b) 105 words typed in 3 min
   c) $2.80 for a 7-min call
   d) $4.74 for 3 kg of oranges
   e) 240 pages printed in 8 min

8. Which toothpaste is the better buy? Show your work.
9. The mass of grass seed needed to seed a yard depends on the area of the yard. Five kilograms of seed cover 100 m².
   a) How much seed is needed for this lawn?
   b) Seed is sold in 8-kg bags. One bag of seed costs $21.20. How much will it cost to seed the lawn in part a?

10. Paige works after school picking apples. It took her 4 days to pick 9 rows of trees.
    a) At this rate, how long will it take her to pick 20 rows of trees? Explain.
    b) Paige picked for 10 days. How many rows did she pick?

11. Mr. O’Shea drove 600 km on 50 L of gas.
    a) How far could he drive on 30 L of gas?
    b) How much gas would he need to travel 420 km?
    c) What strategy did you use to solve each problem? Explain your strategy and why you chose it.

12. A photocopier can print 12 copies in 48 s. At this rate, how many copies can it print in 1 min?

13. Determine each percent.
    a) 20\% of $56.99
    b) 45\% of $118.56
    c) 30\% of $89.99
    d) 25\% of $37.88

14. A winter jacket is regularly priced at $79.99. It is on sale for 35\% off.
    a) What is the sale price?
    b) What does the customer pay, including taxes?

15. The Canadian Radio and Television Commission requires 60\% of the programming on CBC to be Canadian content. The CBC broadcasts from 6 a.m. one day to 2 a.m. the next each day. How many hours of programming each week are Canadian content?

16. Sheila put $350 in a savings account for 10 months. The annual interest rate was 3\%.
    a) How much simple interest did the money earn?
    b) How much money was in the account after 10 months?

17. James borrowed $1500 for 8 months. The annual interest rate was 9\%.
    a) How much simple interest did James pay?
    b) What did the loan cost James?
Multiple Choice: Choose the correct answer for questions 1 and 2.

1. The ratio of length to width of a rectangle is 5 : 3.
   Which dimensions could be those of the rectangle?
   A. 9 cm by 12 cm
   B. 16 km by 10 km
   C. 10 m by 6 m
   D. 9 mm by 16 mm

2. A 4-L can of paint covers an area of 32 m².
   What area will a 10-L can of paint cover?
   A. 40 m²
   B. 60 m²
   C. 80 m²
   D. 320 m²

Show your work for questions 3 to 6.

3. Knowledge and Understanding
   Determine the value of each variable.
   a) \( \frac{2}{10} = \frac{5}{a} \)
   b) \( \frac{30}{42} = \frac{b}{7} \)
   c) \( 18 : 30 = 12 : c \)
   d) \( \frac{n}{4} = \frac{21}{10} \)

4. Communication
   Sun Li bought 12 oranges for $6.48.
   How many oranges could she buy with $10.00?
   How many different ways could you find out?
   Explain each way.

5. Application
   Jenna put $2500 in a savings account.
   The annual interest rate was 2%.
   a) How much simple interest did the money earn in 3 months?
   b) How much money was in the account after 3 months?

6. Thinking
   The sun is shining.
   You have a measuring tape and a friend to help you. How could you determine the height of a flagpole?
   Explain your strategy.
1. Determine the perimeter and area of each figure.
   a) $\begin{array}{c}
   \text{5 m} \\
   \text{13 m}
   \end{array}$
   
   b) $\begin{array}{c}
   \text{3 cm} \\
   \text{7 cm} \\
   \text{5 cm}
   \end{array}$

2. Determine the area and perimeter of this figure.
   The curve is a semicircle.

3. Determine the volume of each object.
   a) $\begin{array}{c}
   \text{5 cm} \\
   \text{6 cm} \\
   \text{9 cm}
   \end{array}$

   b) $\begin{array}{c}
   \text{1 m} \\
   \text{7 m}
   \end{array}$

   c) $\begin{array}{c}
   \text{4 cm} \\
   \text{14 cm}
   \end{array}$

   d) $\begin{array}{c}
   \text{12 cm}
   \end{array}$

4. A pyramid has height 12 cm and base area 28 cm$^2$.
   a) What is the volume of the pyramid?
   b) What is the height of the related prism?
   c) What would the prism’s height have to be for it to have the same volume as the pyramid?
   Check your answer.

5. A rectangle has area 20 cm$^2$.
   Give 4 possible lengths and widths for the rectangle.

6. Kate has 18 m of edging for a rectangular garden.
   a) What are the dimensions of the largest garden Kate can enclose?
   b) What is the area of this garden?
   c) Suppose one side of the garden is along an existing fence and does not need edging.
      i) What are the dimensions of the largest garden that Kate can enclose?
      ii) What is its area?

7. a) For each area, determine the dimensions of a rectangle with the minimum perimeter.
      i) 25 cm$^2$
      ii) 32 cm$^2$
      iii) 78 m$^2$
   b) Calculate the perimeter of each rectangle in part a.
   How do you know each perimeter is a minimum?
8. An isosceles triangle has one angle measuring 34°. What are the measures of the other 2 angles? Give two possible answers.

9. Determine the angle measure indicated by each letter. Justify your answers.
   a) \[ \angle q = 103° \]
   b) \[ \angle y = 30° \]
   c) \[ \angle b = 75° \]

10. a) Use this diagram to determine the sum of the interior angles in a hexagon.

   b) Show a different way that you could divide this hexagon to determine the sum of the interior angles. Determine the sum. How does it compare to your answer to part a?

   c) A regular hexagon has 6 equal sides and 6 equal angles. Determine each measure.
      i) an interior angle
      ii) an exterior angle

11. Write 2 equivalent ratios for each ratio.
    a) \( 2 : 7 \)
    b) \( 45 : 165 \)
    c) \( 21 : 9 \)
    d) \( 100 : 24 \)

12. A recipe that makes 6 dozen meringue drops calls for 4 egg whites and 1 cup of sugar.
    a) Saad has only 3 eggs. How much sugar should he use?
    b) How many meringue drops will Saad make?

13. Hannah’s favourite cereal comes in 2 sizes. Which is the better buy? Show your work.
    - 775 g for $5.49
    - 400 g for $2.97

14. Running an old gas-powered lawn mower for 1 h can create as much pollution as driving a new car 550 km. It takes Isaac 45 min to mow his front and back lawns. How far would a person have to drive a new car to generate the same pollution?

15. Determine each percent.
    a) 30% of $129.99
    b) 15% of $27.54
    c) 40% of $215.49

16. Hong put $550 in a savings account for 9 months. The annual interest rate was 4%.
    a) How much simple interest did the money earn?
    b) How much money was in the account after 9 months?