2 Investigating Perimeter and Area of Rectangles

What You'll Learn

To determine the maximum area of a rectangle for a given perimeter and to determine the minimum perimeter of a rectangle for a given area

And Why

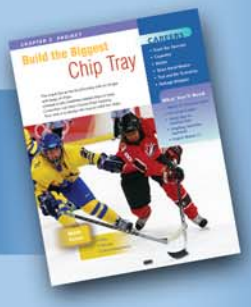
In real-world problems, we often want to maximize space or minimize cost.

Key Words

- maximum
- minimum

Project Link

- Build the Biggest Chip Tray



Reading and Writing in Math

Exploring Guided Examples

All sections of this textbook have one *Guided Example*. A Guided Example is a detailed solution to a problem.

Reading a Guided Example

Reading a Guided Example will help you learn the math in that section.

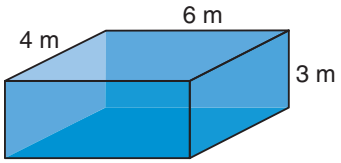
When you read a Guided Example, you should make sure you understand how one step follows from the previous steps.

Example

A rectangular prism has dimensions 6 m by 4 m by 3 m. Determine the volume of the prism.

Solution

Sketch and label a prism.



Use the formula for the volume of a rectangular prism: $V = \ell wh$

Substitute: $\ell = 6, w = 4$, and $h = 3$

$V = 6 \times 4 \times 3$

$= 72$

The volume of the prism is 72 m^3.

Volume is measured in cubic units.

- ➢ Work with a partner. Read the Solution above.
- ➢ Make sure you understand each step in the Solution.
- ➢ Read and explain each step in the Solution to your partner.

Writing a Guided Example

Writing a solution that looks like a Guided Example can help you organize your work and show all your steps.

Use the Guided Example above as a guide.

- ➢ Write a solution to the following problem.

Example

A rectangular prism has dimensions 12 cm by 6 cm by 8 cm. Determine the volume of the prism.

- ➢ Compare your solution with your partner's.
- ➢ Do the solutions look like the Guided Example above? Explain any differences.

2.1 Varying and Fixed Measures

Fiona and Justin have a chocolate bar to share.
The bar is 10 cm long.
Fiona can cut the bar anywhere she chooses.

Investigate Cutting a Fixed Length

➢ When Fiona cuts the bar, what might the lengths of the two pieces be? Copy and complete this table.

Which tools could you use to help you solve this problem?

Length of one piece (cm)	Length of the other piece (cm)

Give at least 6 possible sets of lengths.

➢ How are the two lengths related?

➢ What patterns do you see in the lengths?

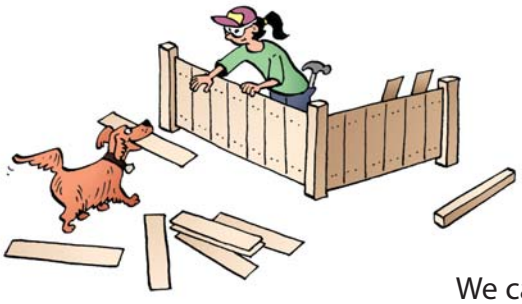
Reflect

➢ Do the lengths have to be whole numbers? Explain.

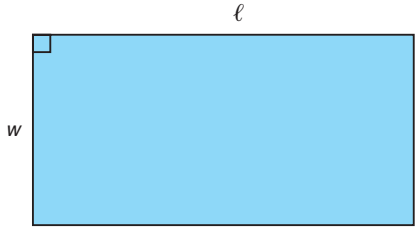
➢ Which measures change?
Which measure stays the same?

➢ What are the lengths likely to be if Justin chooses his piece first?

Connect the Ideas



Nikki is building a rectangular dog pen.
She has 24 m of fence.



We can determine possible dimensions for the pen.

The perimeter of a rectangle is twice the sum of its length and width.

The perimeter is 24 m.
So, the sum of the length and width is 12 m.
$\ell + w = 12$

List pairs of measures whose sum is 12.

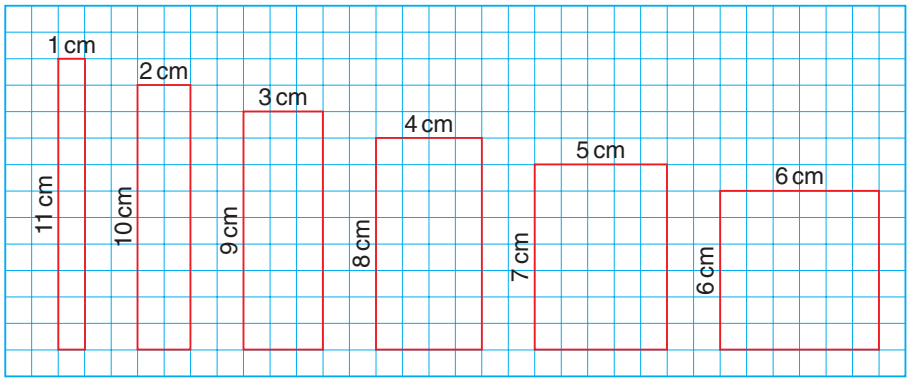
Width (m)	Length (m)	Perimeter (m)
1	11	24
2	10	24
3	9	24
4	8	24
5	7	24
6	6	24

The perimeter stays the same.
The length and width vary.

We do not need to continue the table because we would repeat the rectangles we have.

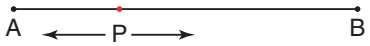
For the perimeter to remain the same, the length decreases as the width increases.
The increase in width is equal to the decrease in length.

The rectangles from the table are drawn on 1-cm grid paper below.



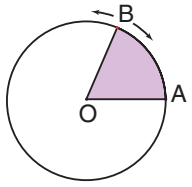
Practice

1. Point P moves along line segment AB.

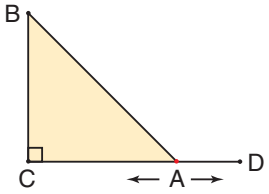


a) Which measures vary?
b) Which measure stays the same?

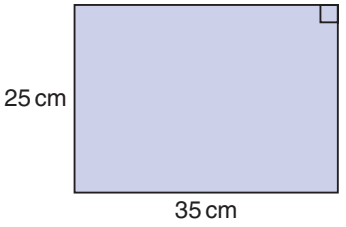
2. Point B moves along the circumference of a circle.
Point O is the centre of the circle.
a) Which measures vary?
b) Which measures stay the same?



3. Point A moves along line segment CD.
Triangle ABC is a right triangle.
In ΔABC:
a) Which measures vary?
b) Which measures stay the same?



4. **a)** Determine the perimeter and area of this rectangle.
b) Suppose the length is increased by 5 cm and the width is decreased by 5 cm.
How do you think the perimeter and area will change?
Explain. Include a diagram in your explanation.
c) Calculate the perimeter and area of the new rectangle.
Were you correct in part b?
Justify your answer.



5. Use the rectangle from question 4.
a) Suppose the length is doubled and the width is halved.
How do you think the perimeter and area will change?
Explain.
b) Calculate the perimeter and area of the new rectangle.
Were you correct in part a?
Justify your answer.
Include a diagram.

6. Sketch 5 different rectangles with each perimeter.
Label each rectangle with its dimensions.
a) Perimeter: 20 units
b) Perimeter: 32 units

Which tools could you use to help you solve this problem?

Example

A rectangle has area 12 cm^2.
Determine 3 different lengths and widths for the rectangle.

Solution

The area of a rectangle is: $A = \ell w$
So, the length times the width is 12 cm^2.
List pairs of measures whose product is 12 cm^2.

Recall that ℓw means $\ell \times w$

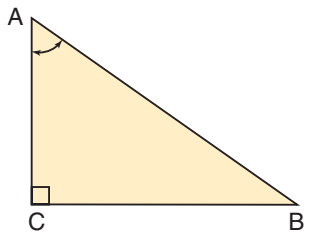
Width (cm)	Length (cm)	Area (cm^2)
1	12	12
2	6	12
3	4	12

7. Sketch 5 different rectangles with each area.
Label each rectangle with its dimensions.
a) Area: 36 square units **b)** Area: 48 square units

8. **Assessment Focus**
a) Give 4 different pairs of lengths and widths for a rectangle with perimeter 16 cm.
b) Do rectangles with the same perimeter have the same area?
c) Give 3 different pairs of lengths and widths for a rectangle with area 16 cm^2.
d) Do rectangles with the same area have the same perimeter?
Justify your answers.

9. **a)** How can the area of a rectangle stay the same when its length and width change?
b) How do you determine possible lengths and widths for a given area?

10. **Take It Further** Triangle ABC is a right triangle.
Sketch this triangle on grid paper.
Suppose the measure of ∠A changes.
a) Which other measures change?
b) Which measures stay the same?
Draw triangles to illustrate your answers.



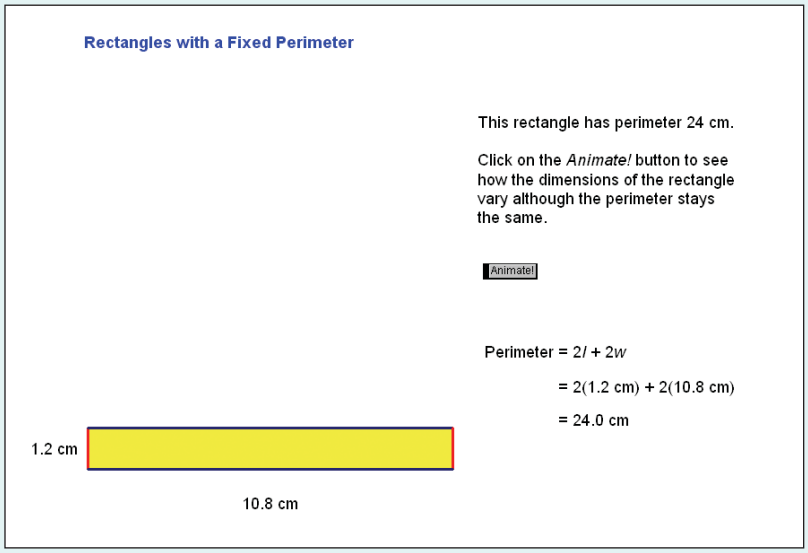
In Your Own Words

Explain how the perimeter of a rectangle can stay the same when the length and width change.
How do you determine the possible lengths and widths for a given perimeter?

Using *The Geometer's Sketchpad* to Investigate Rectangles

Part 1: Rectangles with a Given Perimeter

1. Open the file *FixedPerimArea.gsp*.
Click the tab for page 1.



2. The rectangle in the sketch has perimeter 24 cm.
Click ***Animate!*** to see how the dimensions of the rectangle vary while the perimeter stays the same.

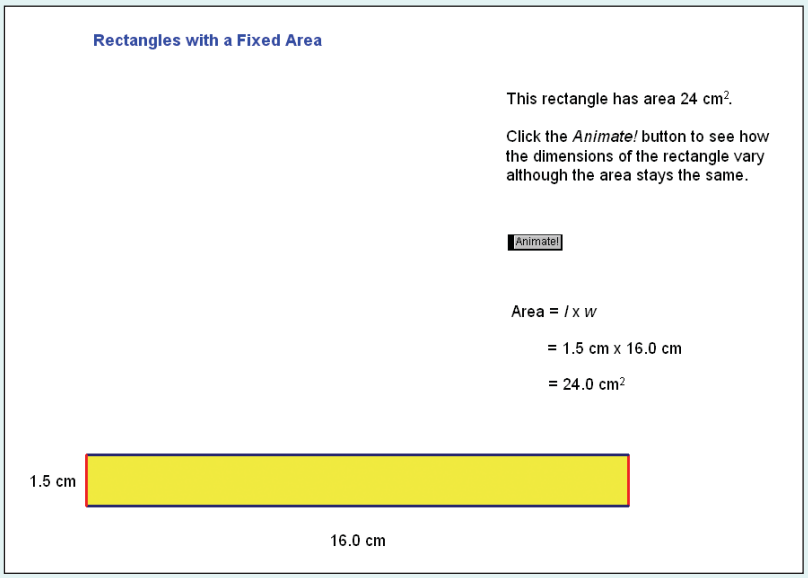
3. a) Click ***Animate!*** to stop the animation.
b) Create this table in your notebook.

Length (cm)	Width (cm)	Perimeter (cm)
		24

c) Start and stop the animation 6 times.
Each time you stop the animation, record the length and width of the rectangle in the table.
d) Describe the patterns in the lengths and widths.
e) Explain how the dimensions of a rectangle can vary while the perimeter stays the same.

Part 2: Rectangles with a Given Area

1. Use the file *FixedPerimArea.gsp*.
Click the tab for page 2.



2. The rectangle in the sketch has area 24 cm^2.
Click ***Animate!*** to see how the dimensions of the rectangle vary while the area stays the same.

3. a) Click ***Animate!*** to stop the animation.
b) Create this table in your notebook.

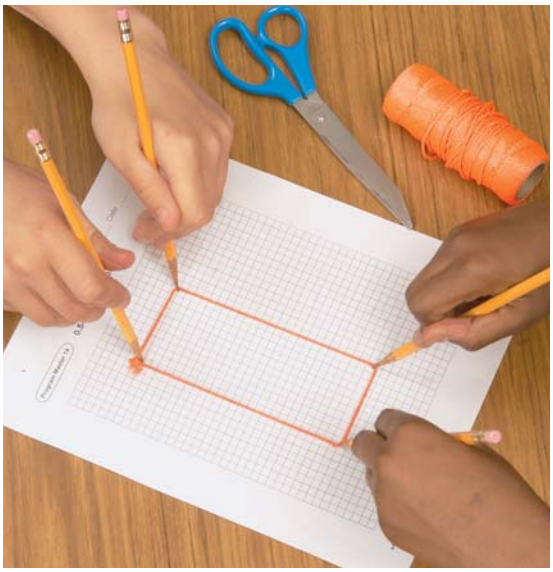
Length (cm)	Width (cm)	Area (cm^2)
		24

c) Start and stop the animation several times.
Each time you stop the animation, record the length and width of the rectangle in the table.
d) Describe the patterns in the lengths and widths.
e) Explain how the dimensions of a rectangle can vary while the area stays the same.

2.2 Rectangles with Given Perimeter or Area

Investigate Rectangles with a Given Perimeter

Work with a partner.
You will need 0.5-cm grid paper.
Your teacher will give you a loop of string.
How many different rectangles can you make with the loop of string?



- Lay the loop of string on grid paper. Place the tips of four pencils inside the loop.
- Use the pencils to make the string taut and form a rectangle. Mark the vertices on the grid paper.

Use the lines on the grid paper to make the angles as close to 90° as possible.

- Remove the string. Use a ruler to join the vertices and draw the rectangle.
  Measure the length and width of the rectangle.
  Calculate its area and perimeter.
  Record these measures.

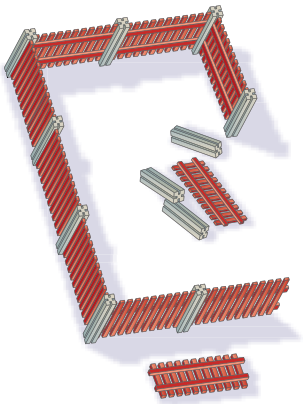
Length (cm)	Width (cm)	Area (cm^2)	Perimeter (cm)

- Use the same loop.
  Repeat the previous steps to create 4 different rectangles.

Reflect

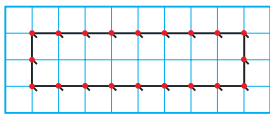
- How do you know when the figure is a rectangle?
- Why do all the rectangles have the same perimeter?
- Why do the rectangles have different areas?

Connect the Ideas

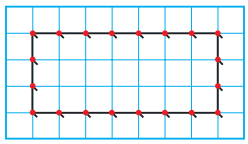


A toy farm set has 20 straight sections of fence.
Each section is 10 cm long.
Here are 2 possible rectangular toy corrals.

1st corral
8 sections long
2 sections wide



2nd corral
7 sections long
3 sections wide



Determine the dimensions

1st corral
$\ell = 8 \times 10$ cm
$= 80$ cm
$w = 2 \times 10$ cm
$= 20$ cm
The corral is 80 cm by 20 cm.

2nd corral
$\ell = 7 \times 10$ cm
$= 70$ cm
$w = 3 \times 10$ cm
$= 30$ cm
The corral is 70 cm by 30 cm.

We can compare the perimeters of the corrals and their areas.

Determine the perimeters

Use the formula: $P = 2\ell + 2w$

1st corral
Substitute: $\ell = 80$ and $w = 20$
$P = 2(80) + 2(20)$
$= 160 + 40$
$= 200$
The perimeter is 200 cm.

Recall that 2(80) means 2×80.

2nd corral
Substitute: $\ell = 70$ and $w = 30$
$P = 2(70) + 2(30)$
$= 140 + 60$
$= 200$
The perimeter is 200 cm.

Determine the areas

Use the formula: $A = \ell w$

1st corral
Substitute: $\ell = 80$ and $w = 20$
$A = (80)(20)$
$= 1600$
The area is 1600 cm^2.

2nd corral
Substitute: $\ell = 70$ and $w = 30$
$A = (70)(30)$
$= 2100$
The area is 2100 cm^2.

The corrals have the same perimeter but different areas.

Practice

1. **a)** Draw each rectangle on grid paper.

i) $\ell = 8, w = 2$ **ii)** $\ell = 8, w = 3$ **iii)** $\ell = 6, w = 4$ **iv)** $\ell = 9, w = 2$

b) Which rectangles in part a have the same perimeter? How do you know?

c) Which rectangles in part a have the same area? How do you know?

d) Can rectangles with the same perimeter have different areas? Explain.

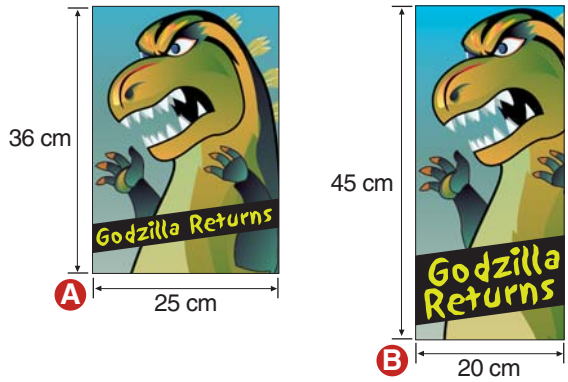
e) Can rectangles with the same area have different perimeters? Explain.

Example

Jacy wants to frame these pictures.

a) Show that both pictures have the same area.

b) Is the same length of frame needed for each picture? Explain.



Solution

a) Use the formula: $A = \ell w$

Picture A

Substitute: $\ell = 36$ and $w = 25$

$A = (36)(25)$

$= 900$

Both pictures have area 900 cm^2.

Picture B

Substitute: $\ell = 45$ and $w = 20$

$A = (45)(20)$

$= 900$

Calculate the perimeter of each picture to determine the length of frame.

b) Use the formula: $P = 2\ell + 2w$

Picture A

Substitute: $\ell = 36$ and $w = 25$

$P = 2(36) + 2(25)$

$= 122$

Picture A needs 122 cm of frame.

So, picture B needs more frame.

Picture B

Substitute: $\ell = 45$ and $w = 20$

$P = 2(45) + 2(20)$

$= 130$

Picture B needs 130 cm of frame.

2. **a)** Draw this rectangle on grid paper.

Draw 5 different rectangles with the same perimeter as this rectangle.

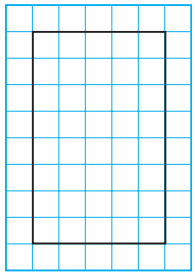
b) Calculate the area of each rectangle.

Write the area inside each rectangle.

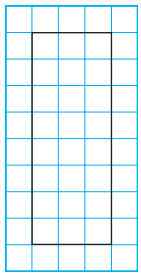
$A = \ell w$

c) Cut out the rectangles. Sort them from least to greatest area.

d) Describe how the shapes of the rectangles change as the area increases.



3. a) Draw this rectangle on grid paper.
Draw 3 different rectangles with the same area as this rectangle.



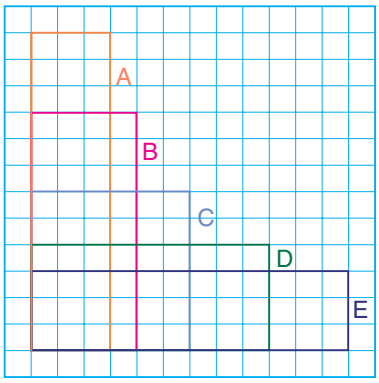
b) Calculate the perimeter of each rectangle.
Write the perimeter inside each rectangle.

$P = 2\ell + 2w$

c) Cut out the rectangles. Sort them from least to greatest perimeter.
d) Describe how the rectangles change as the perimeter increases.

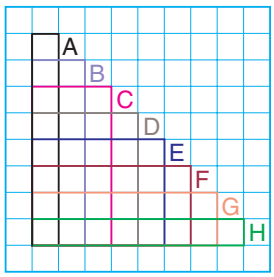
4. **Assessment Focus**
a) Draw a square on grid paper.
Determine its area and perimeter.
b) Draw a rectangle with the same perimeter but a different area.
c) Which figure has the greater area? Show your work.

5. This diagram shows overlapping rectangles drawn on 1-cm grid paper.
All of them share the same vertex.



a) Calculate the area of each rectangle.
What do you notice?
b) Which rectangle do you think has the least perimeter?
The greatest perimeter? Justify your choice.
c) Calculate the perimeter of each rectangle.
Were you correct in part b? Explain.

6. This diagram shows overlapping rectangles drawn on 1-cm grid paper. All of them share the same vertex.



a) Calculate the perimeter of each rectangle.
What do you notice?
b) Which rectangle do you think has the least area?
The greatest area? Justify your choice.
c) Calculate the area of each rectangle.
Were you correct in part b? Explain.

7. **Take It Further** Can two different rectangles have the same perimeter and the same area? Justify your answer.

In Your Own Words

Explain how rectangles can have the same perimeter but different areas.
Explain how rectangles can have the same area but different perimeters.
Include diagrams in your explanations.

2.3 Maximum Area for a Given Perimeter

A building supply store donates fencing to a day-care centre.
The fencing will be used to create a rectangular play area.
Which dimensions will give the largest possible area?

Investigate Greatest Area of a Rectangle

Work in a group of 4.
You will need a 10 by 10 geoboard and geobands.

- Choose one of these perimeters:
  10 units, 16 units, 18 units, or 20 units
- Make as many different rectangles as you can with that perimeter.
  Record the length and width of each rectangle in a table.
  Determine the area of each rectangle.

Length (units)	Width (units)	Area (square units)

- Which rectangle has the greatest area?

Reflect

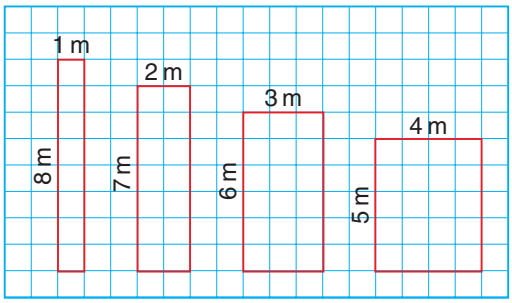
- How do you know you found the rectangle with the greatest area?
- What is the greatest area?
  Describe the rectangle with the greatest area.
- Share results with your group.
  Compare the rectangles with the greatest area for a given perimeter.
  How are they the same? How are they different?

Connect the Ideas

Since the edging cannot be cut, the dimensions of the rectangles must be whole numbers.

Maya has 18 m of edging to create a rectangular flowerbed. What dimensions will give the greatest area?

➢ Suppose the edging is in 1-m sections and cannot be cut. Draw rectangles to represent flowerbeds with perimeter 18 m. Calculate the area of each rectangle.

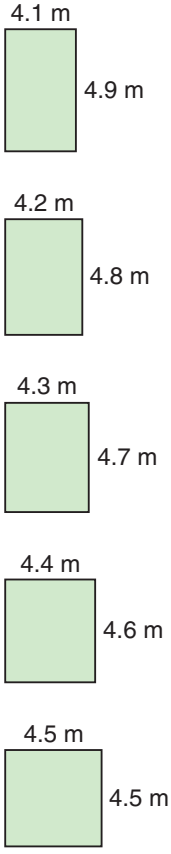


Length (m)	Width (m)	Area (m^2)
8	1	8
7	2	14
6	3	18
5	**4**	**20**

From the table, the greatest or **maximum** area is 20 m^2.

The greatest area occurs when the dimensions of the flowerbed are closest in value: 5 m and 4 m

➢ If the edging can be cut, Maya can make flowerbeds with dimensions that are even closer in value.



Length (m)	Width (m)	Area (m^2)
4.9	4.1	20.09
4.8	4.2	20.16
4.7	4.3	20.21
4.6	4.4	20.24
4.5	**4.5**	**20.25**

From the table, the maximum area is 20.25 m^2.

This maximum area occurs when the dimensions of the flowerbed are equal.

That is, the rectangle is a square with side length 4.5 cm.

This result is true in general. Among all rectangles with a given perimeter, a square has the maximum area.

It may not be possible to form a square when the sides of the rectangle are restricted to whole numbers.
Then, the maximum area occurs when the length and width are closest in value.

Practice

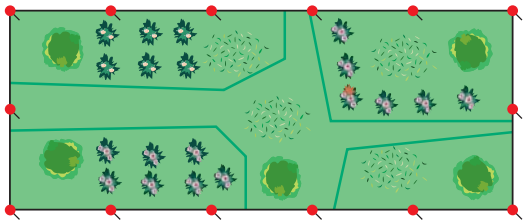
1. **a)** On 1-cm grid paper, draw all possible rectangles with each perimeter. Each dimension is a whole number of centimetres.

i) 8 cm **ii)** 12 cm **iii)** 22 cm

b) Calculate the area of each rectangle in part a.

c) For each perimeter in part a, what are the dimensions of the rectangle with the maximum area?

2. Suppose you have 14 sections of fence to enclose a rectangular garden.



Each section is 1 m long and cannot be cut.

a) Sketch the possible gardens that can be enclosed by the fence. Label each garden with its dimensions.

b) Calculate the area of each garden in part a.

c) Which garden has the maximum area? Is this garden a square? Explain.

Example

What is the maximum area of a rectangle with perimeter 30 m?

Solution

The maximum area occurs when the rectangle is a square.

Determine the side length, s, of a square with perimeter 30 m.

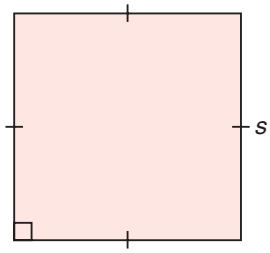
Use the formula: $P = 4s$

Substitute: $P = 30$

$30 = 4s$

Think: What do we multiply 4 by to get 30?

Divide 30 by 4 to find out.



$s = \frac{30}{4}$

$s = 7.5$

Calculate the area of the square. Use the formula: $A = s^2$

Substitute: $s = 7.5$

$A = (7.5)^2$

$= 56.25$

The maximum area is 56.25 m^2.

3. What is the maximum area for a rectangle with each perimeter? Explain your thinking.

a) 36 cm **b)** 60 cm **c)** 75 cm

Need Help?
Read the Example on page 55.

4. **Assessment Focus** Determine the dimensions of a rectangle with perimeter 42 m whose area is as great as possible. What is the maximum area? Justify your answer.

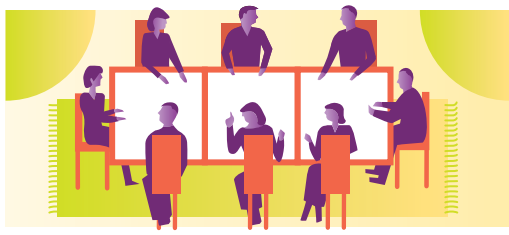
5. Joachim has been comparing rectangles that have the same perimeter. He says that rectangles whose lengths and widths are close in value have larger areas than rectangles whose lengths and widths are very different. Do you agree with Joachim? Justify your answer. Include diagrams and calculations.

6. In a banquet room, there are small square tables that seat 1 person on each side. These tables are pushed together to create a rectangular table that seats 20 people.

a) Consider all possible arrangements of square tables to seat 20 people. Sketch each arrangement.

b) Which arrangement requires the most tables? The fewest tables?

c) Explain why the arrangement with the fewest tables might not be preferred in this situation.



Three tables seat 8 people.

Which tools could you use to help you solve this problem?

7. **Take It Further** Lisa and her sister Bonnie are each given 8 m of plastic edging to create a flowerbed. Lisa creates a square flowerbed. Bonnie creates a circular flowerbed.

a) Calculate the area of each girl's flowerbed.

b) Which figure encloses the greater area? Do you think the areas of all squares and circles with the same perimeter are related this way? Draw other squares and circles to find out.

Use *The Geometer's Sketchpad* if available.

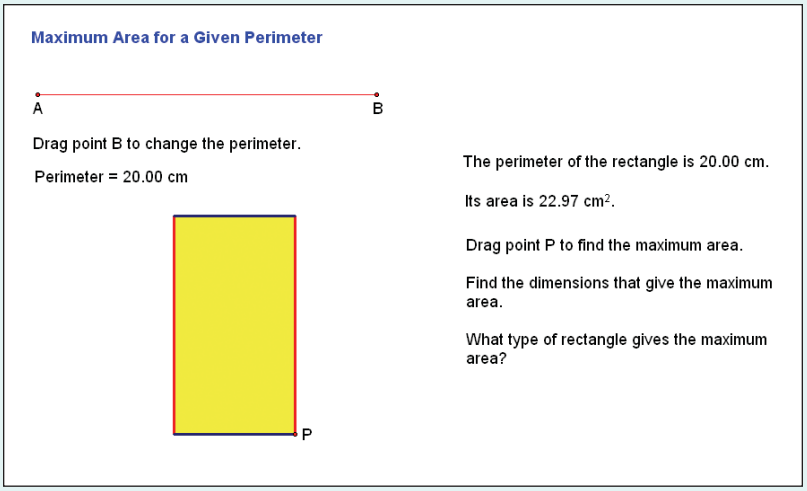
In Your Own Words

Suppose your friend was absent during this lesson. Describe what you learned. Include diagrams and examples in your explanation.

Using *The Geometer's Sketchpad* to Investigate Maximum Area and Minimum Perimeter

Part 1: Maximum Area

1. Open the file *MaxArea.gsp*.



2. The rectangle in the sketch has perimeter 20 cm. Drag point P to see how the area of the rectangle varies while the perimeter stays the same.

a) What is the maximum area of the rectangle?

b) Measure the length and width of the rectangle with maximum area. What do you notice?

c) Record your results in a table.

Your teacher will tell you how to measure with *The Geometer's Sketchpad*.

Perimeter (cm)	Maximum area (cm^2)	Length (cm)	Width (cm)
20			

3. **a)** Drag point B to create a rectangle with a different perimeter.

b) Determine the maximum area of the rectangle.

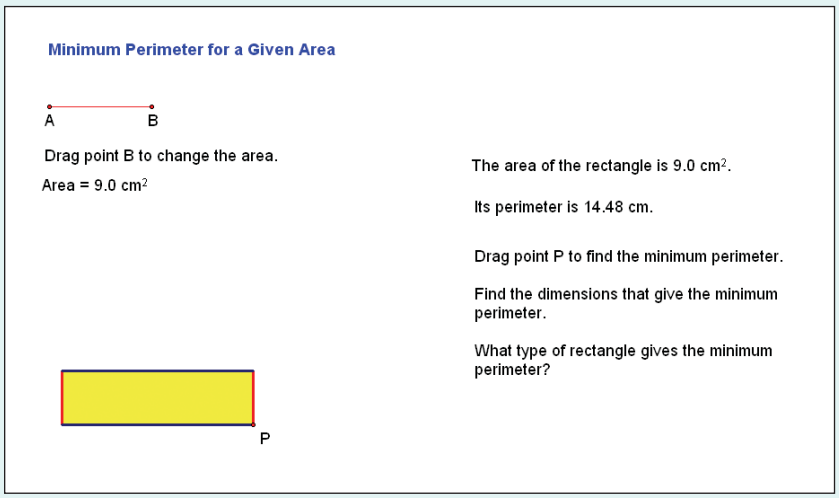
c) Record your results in the table.

4. Repeat the steps in question 3 for 3 more rectangles.

5. Describe the patterns in the table.

Part 2: Minimum Perimeter

1. Open the file *MinPerim.gsp*.



2. The rectangle in the sketch has area 9 cm^2.
Drag point P to see how the perimeter of the rectangle varies while the area stays the same.

a) What is the minimum perimeter of the rectangle?

b) Measure the length and width of the rectangle with minimum perimeter.
What do you notice?

c) Record your results in a table.

Area (cm^2)	Minimum perimeter (cm)	Length (cm)	Width (cm)
9			

3. **a)** Drag point B to create a rectangle with a different area.

b) Determine the minimum perimeter of the rectangle.

c) Record your results in the table.

4. Repeat the steps in question 3 for 2 more rectangles.

5. Describe the patterns in the table.

Find the Least Perimeter

GAME

Materials:

- deck of playing cards

- number cube labelled 16, 20, 24, 28, 32, 36

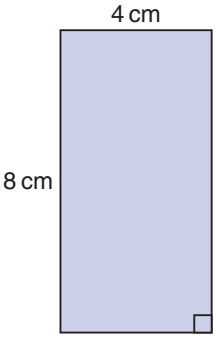
The numbers on the cube are the areas of rectangles.

Play with a classmate.

- ➢ Player B deals 6 cards to each player.
  He places the other cards face down.
- ➢ Player A rolls the number cube.
  The number rolled represents the area of a rectangle.
- ➢ Player B finds 2 cards that are the length and width of a rectangle with that area.
  Player B states the perimeter of the rectangle.
  If he is correct, he gets 1 point.
  If it is the least perimeter for that rectangle, he gets 2 points.
  Player B puts the cards in a discard pile and takes 2 more cards from the remaining deck.
  Then Player B rolls the cube for Player A to find the cards.
- ➢ If Player B does not have the 2 cards, he can take a card from the remaining deck.
  If Player B still does not have the 2 cards he needs, Player A tries to find the 2 cards.
  If no one has the two cards, Player A rolls the cube again.
- ➢ Play continues until no cards remain in the deck.
  The person with more points wins.

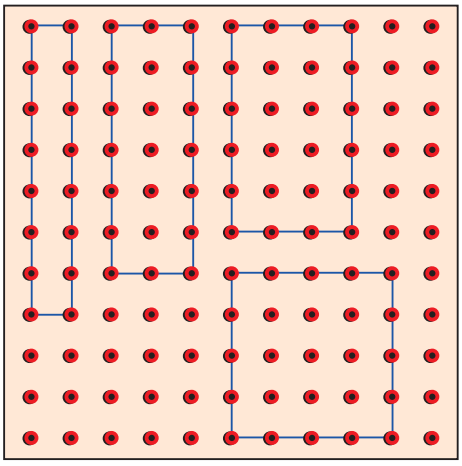
Mid-Chapter Review

2.1 **1.** Explain how the area and perimeter of the rectangle changes in each scenario.



a) The length stays the same but the width increases.
b) The length stays the same but the width decreases.
c) Both the length and width increase.
d) Both the length and width decrease.
e) The length decreases and the width increases.
The amount of decrease equals the amount of increase.

2. Alat made these rectangles on a geoboard.



a) What is the same about all these rectangles?
b) What is different?

2.2 **3.** **a)** List 4 possible lengths and widths for a rectangle with each area.
i) 30 m^2 **ii)** 64 m^2
b) Determine the perimeter of each rectangle in part a.

4. A rectangle is 15 cm long and 9 cm wide. Its length and width change as described below.
a) The length increases by 3 cm and the width decreases by 3 cm.
b) The length is multiplied by 3 and the width is divided by 3.
In each case, does the area change? Does the perimeter change? Explain. Include diagrams and calculations in your explanation.

5. A farmer has 2 horse paddocks.
One paddock is 10 m by 90 m.
The other paddock is 30 m by 30 m.
a) Show that both paddocks have the same area.
b) Is the same amount of fencing required to enclose each paddock? Explain.

2.3 **6.** Draw these rectangles on grid paper.
Rectangle A: 20 cm by 10 cm
Rectangle B: Same perimeter as rectangle A but smaller area
Rectangle C: Same perimeter as rectangle A and the greatest possible area
Write the dimensions, perimeter, and area of each rectangle.

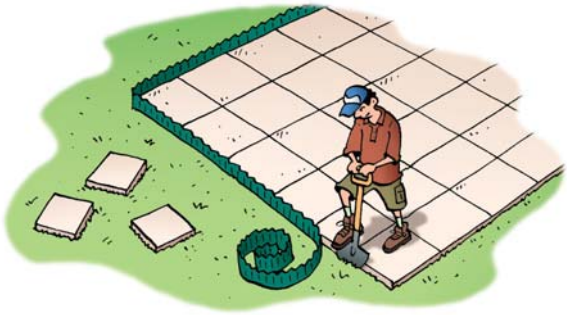
7. What is the maximum area of a rectangular lot that can be enclosed by 180 m of fencing?

2.4 Minimum Perimeter for a Given Area

Miriam works in a garden centre.
She is asked what the least amount of edging would be for a certain rectangular area of grass.
How might she find out?

Investigate Least Edging for a Patio

Michael has 36 square stones to arrange as a rectangular patio.
He will then buy edging to go around the patio.



- Sketch the different patios Michael can create.
  Label each patio with its dimensions.
- Which patio requires the least amount of edging?
- Suppose each stone has side length 30 cm.
  What is the least amount of edging needed for the patio?
  Justify your answer.

Which tools could you use to help you solve this problem?

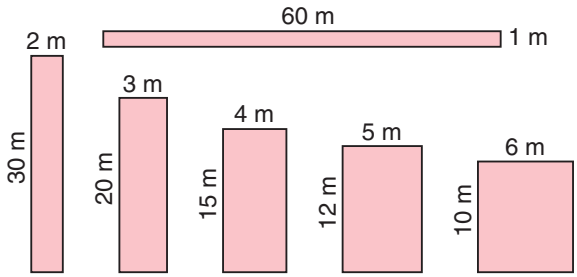
Reflect

- Is a 9 by 4 arrangement the same as a 4 by 9 arrangement? Explain.
- Which measure is the same for all the patios?
  Which measures are different?
- What assumptions or estimates did you make to calculate the amount of edging?
- Describe the rectangle that uses the least amount of edging.

Connect the Ideas

➢ Eric plans to build a storage shed with floor area 60 m^2.
He wants to use the least amount of materials.
So, Eric needs the least perimeter for the floor.
He will use 60 square patio stones, each with area 1 m^2.

Here are some rectangles that represent floors with area 60 m^2.



The rectangles are not drawn to scale.

Here are some possible dimensions and perimeters of the floor.

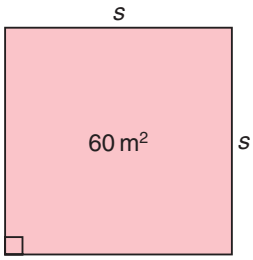
Length (m)	Width (m)	Perimeter (m)
60	1	122
30	2	64
20	3	46
15	4	38
12	5	34
10	**6**	**32**

We do not need to continue the table because we would repeat the rectangles we have.

From the table, the least or **minimum** perimeter is 32 m.

The minimum perimeter occurs when the dimensions of the floor are closest in value: 10 m and 6 m

➢ Jenna suggests that Eric could have a lesser perimeter if he used concrete instead of patio stones.
Jenna drew a square with area 60 m^2.

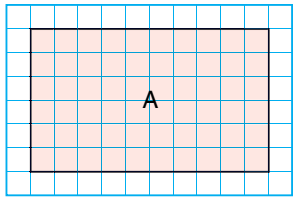


The area of the square is $s \times s$. The area is also 60 m^2.
To find two equal numbers whose product is 60,
Jenna used her calculator to determine the square root of 60.
$\sqrt{60} \doteq 7.75$
A square with side length 7.75 m has perimeter:
$4 \times 7.75\text{ m} = 31\text{ m}$

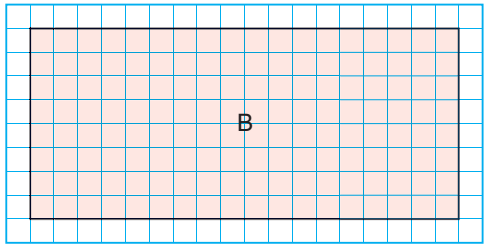
This is true in general. Among all rectangles with a given area, a square has the minimum perimeter.

It may not be possible to form a square when the sides of the rectangle are restricted to whole numbers.
Then, the minimum perimeter occurs when the length and width are closest in value.

Practice



1. **a)** Draw Rectangle A on grid paper.
b) Draw a different rectangle that has the same area as Rectangle A.
c) Do the two rectangles have the same perimeter? Explain.



2. **a)** Draw Rectangle B on grid paper.
b) Draw a different rectangle that has the same area as Rectangle B, but a lesser perimeter.
c) Draw a rectangle that has the same area as Rectangle B with the minimum perimeter.

3. **a)** For each number of congruent square patio stones, which arrangement would give the minimum perimeter?
i) 25 stones **ii)** 50 stones **iii)** 75 stones **iv)** 100 stones
b) Suppose each stone in part a has side length 30 cm. What is the minimum perimeter?

Remember that congruent squares are the same size.

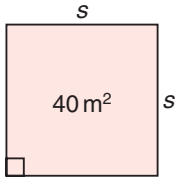
4. Look at the areas and perimeters in question 3.
a) What patterns do you see in the areas?
b) Do the perimeters have the same pattern? Explain.

Example

What is the minimum perimeter for a rectangle with area 40 m^2?

Solution

The minimum perimeter occurs when the rectangle is a square.
Determine the side length, s, of a square with area 40 m^2.



Use the formula: $A = s^2$
Substitute: $A = 40$
$40 = s^2$
$s = \sqrt{40}$
$s \doteq 6.325$
Calculate the perimeter of the square.
Use the formula: $A = 4s$
Substitute: $s = 6.325$
$P = 4(6.325)$
$= 25.3$
The minimum perimeter for the rectangle is approximately 25.3 m.

In questions 5 to 7, round your answers to 1 decimal place.

5. What is the minimum perimeter for a rectangle with each area?

a) 30 m^2 **b)** 60 m^2 **c)** 90 m^2 **d)** 120 m^2

6. **Assessment Focus** Determine the dimensions of a rectangle with area 1000 m^2 whose perimeter is the least possible. What is the minimum perimeter? Justify your answer.

Having Trouble?
Read Connect the Ideas.

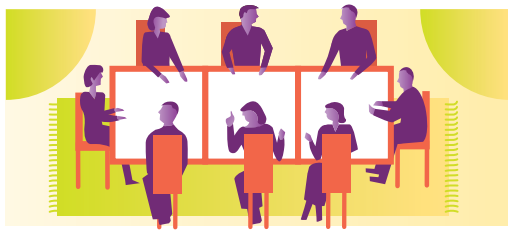
7. A rectangular patio is to be built from 56 congruent square tiles.

a) Which patio design will give the minimum perimeter?

b) Is this patio a square? Explain.

8. Keung has been comparing rectangles that have the same area. He says that rectangles whose lengths and widths are close in value have greater perimeters than rectangles whose lengths and widths are very different. Do you agree? Explain.

9. In a banquet room, there are small square tables that seat 1 person on each side. The tables are pushed together to create larger rectangular tables.



a) Consider all possible arrangements of 12 square tables. Sketch each arrangement on grid paper.

b) Which arrangement seats the most people? The fewest people? Explain.

c) Explain why the minimum perimeter might not be preferred in this situation.

10. **Take It Further**

a) Draw a square.

b) Draw a rectangle that has the same area as the square but a different perimeter.

c) Is it possible to draw a rectangle that has the same area as the square but a lesser perimeter than the square? Explain.

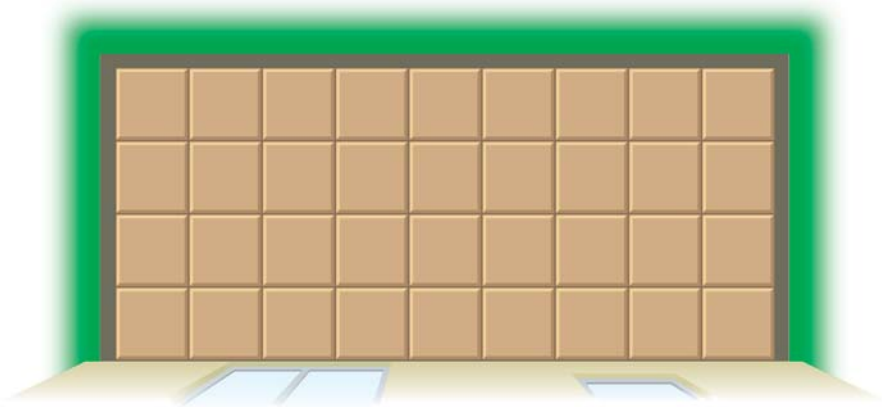
Which tools could you use to solve this problem?

In Your Own Words

For a given area, describe the rectangle that has the least perimeter. Will the rectangle always be a square? Explain.

2.5 Problems Involving Maximum or Minimum Measures

Recall, from page 61, that Michael was building a rectangular patio with 36 square stones. To save money on edging, Michael decides to place the patio against the side of the house. Now, edging will be needed on only three sides of the patio.



Investigate Solving a Problem About Minimum Length

Which tools could you use to solve this problem?

For Michael's patio:

- What should the dimensions be so the patio uses the minimum length of edging?
- Each stone has side length 30 cm. What is the minimum length of edging? Justify your answer.

Reflect

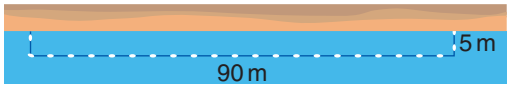
- Could you have used the patios you drew in Section 2.4 *Investigate*? Explain.
- Will a 4 by 9 patio and a 9 by 4 patio require the same length of edging? Explain.
- Is the patio that requires the minimum length of edging a square? Explain.

Connect the Ideas

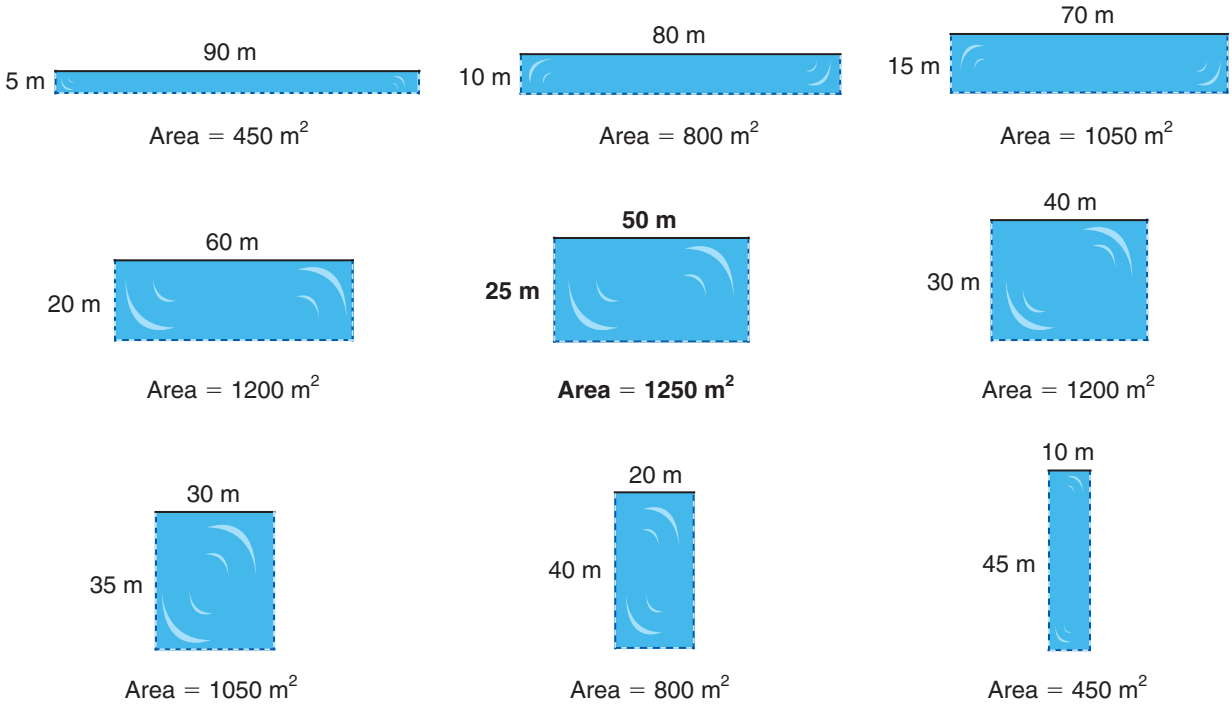
A rectangular swimming area is to be enclosed by 100 m of rope.
One side of the swimming area is along the shore,
so the rope will only be used on three sides.
What is the maximum swimming area that can be created?

Sketch rectangles to represent the swimming areas.
Calculate the area of each rectangle.

For example, consider a swimming area with width 5 m.



Two widths and 1 length are formed by the rope.
The rope is 100 m.
Two widths are: 2(5 m) = 10 m
So, the length is: 100 m – 10 m = 90 m
The area is: (5 m)(90 m) = 450 m^2



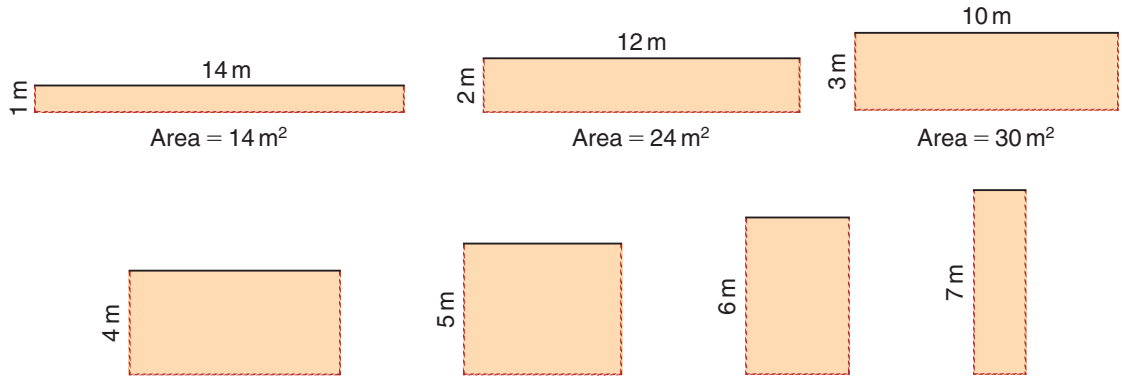
The areas increase, then decrease.
The maximum swimming area is 1250 m^2.
Only one of the rectangles has this area.

The maximum swimming area is not a square.
It is a rectangle that measures 50 m by 25 m.

Practice

1. A store owner uses 16 m of rope for a rectangular display. There is a wall on one side, so the rope is only used for the other 3 sides.

a) Copy these rectangles. Determine the missing measures.



b) What are the dimensions of the maximum area that can be enclosed? How are these dimensions related?

2. Cody has 20 m of fencing to build a rectangular pen for his dog. One side of the pen will be against his house.

a) Sketch and label as many possible rectangles as you can.

b) Copy and complete this table.

Width (m)	Length (m)	Area (m^2)
1	18	18

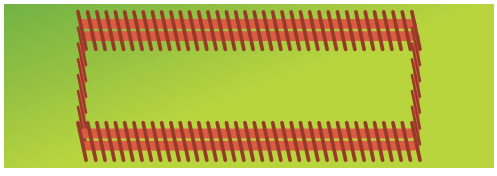
Continue until the width is 9 m.

c) What dimensions give the maximum area for the pen? How are these dimensions related?

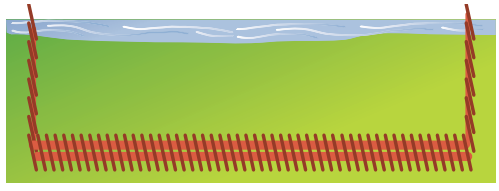
3. **Assessment Focus** A rectangular field is enclosed by 80 m of fencing.

a) What are the dimensions of the field with maximum area in each situation?

i) The fence encloses the entire field.



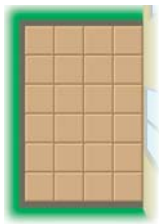
ii) One side of the field is not fenced.



b) How are the dimensions of the rectangles in part a related?

Example

A patio is to be built on the side of a house using 24 congruent square stones. It will then be edged on 3 sides. Which arrangement requires the minimum edging?



Solution

The area of the patio is 24 square units.
List pairs of numbers with product 24.
Calculate the edging required for each patio.

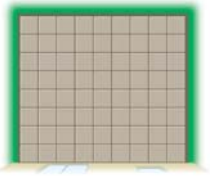
Width (units)	Length (units)	Edging (units)
1	24	26
2	12	16
3	**8**	**14**
4	**6**	**14**
6	4	16
8	3	19
12	2	26
24	1	49

For a 3 by 8 arrangement, this is:

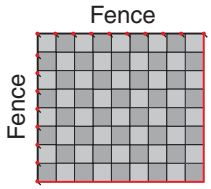
$$\text{Amount of edging} = 2(3) + 8$$
$$= 14$$

A 4 by 6 arrangement uses the same amount of edging. So, there are 2 different arrangements that require the minimum edging.

4. A patio of 72 congruent square stones is to be built on the side of a house. Sketch and label possible patio designs. Which design has the minimum distance around the sides not attached to the house?



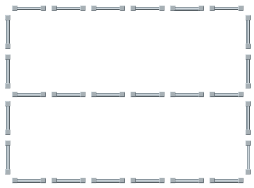
5. A patio of 72 congruent square stones is to be built in the corner of a backyard against a fence. Sketch and label possible patio designs. Which design requires the minimum edging for the sides not against the fence?



6. **Take It Further** Twenty-six sections of fence, each 1 m long, are used to create two storage enclosures of equal area.

a) Find another way to arrange 26 sections to make two equal storage areas.
The storage areas should share one side.

b) What should the length and width of each storage area be to make the enclosure with the maximum area? What are the dimensions of this enclosure?



Which tools could you use to solve this problem?

In Your Own Words

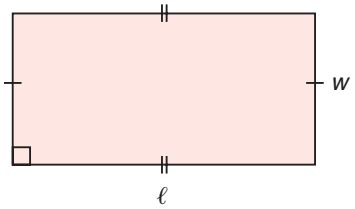
For a given area, you have found the rectangle with the minimum perimeter and the rectangle with the minimum length of 3 sides. How are the rectangles different? Include sketches in your explanation.

Chapter Review

What Do I Need to Know?

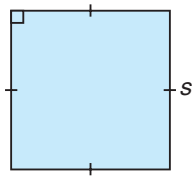
Area and Perimeter Formulas

Rectangle



Area: $A = \ell w$
Perimeter: $P = 2(\ell + w)$
or, $P = 2\ell + 2w$

Square

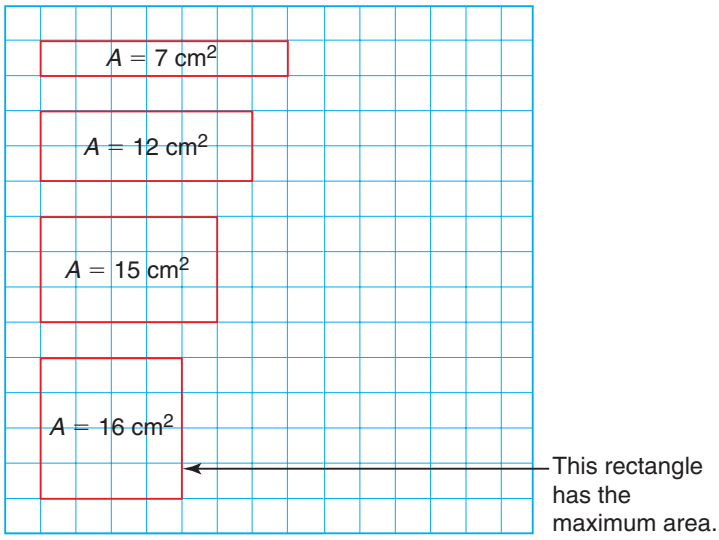


Area: $A = s^2$
Perimeter: $P = 4s$

Maximum Area of a Rectangle

Rectangles with the same perimeter can have different areas.
For example, all these rectangles have perimeter 16 cm.

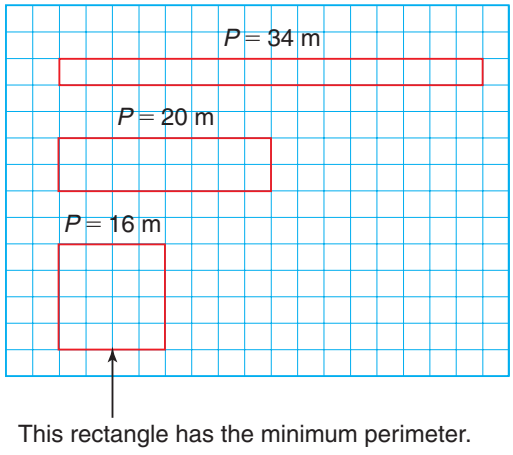
If the dimensions of the rectangle are restricted to whole numbers, it may not be possible to form a square.
The maximum area occurs when the length and width are closest in value.



Minimum Perimeter of a Rectangle

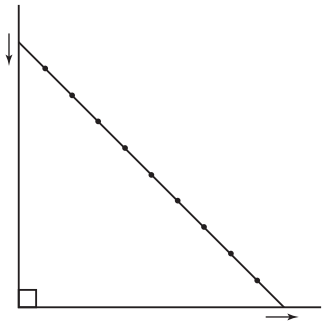
Rectangles with the same area can have different perimeters.
For example, all these rectangles have area 16 m².

If the rectangle is made from congruent square tiles, it may not be possible to form a square.
The minimum perimeter occurs when the length and width are closest in value.



What Should I Be Able to Do?

2.1 **1.** A 3-m ladder leans against a wall. The ladder slips down the wall.



a) Which measures stay the same?
b) Which measures vary? Tell whether each measure is increasing or decreasing.

2.2 **2.** A rectangle has perimeter 100 cm.
a) Copy and complete this table. For each width, determine the length and area of the rectangle.

Width (cm)	Length (cm)	Area (cm^2)
5		
10		
15		
20		
25		

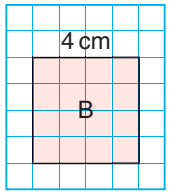
b) Describe the patterns in the table.

3. A rectangle has area 72 cm^2.
a) Copy and complete this table. For each width, determine the length and perimeter of the rectangle.

Width (cm)	Length (cm)	Perimeter (cm)
2		
4		
6		
8		

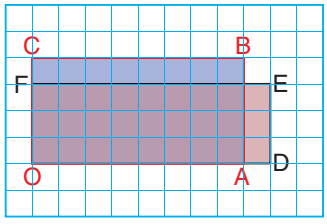
b) Describe the patterns in the table.

4. a) Determine the perimeter and area of square B.



b) Draw a rectangle that has the same perimeter as square B, but a different area.
c) Can you draw a rectangle that has the same perimeter as square B, but a greater area? Justify your answer.

2.3 **5. a)** Do rectangle OABC and rectangle ODEF have the same perimeter? The same area? Explain.



b) Copy the rectangles in part a on grid paper. On the same diagram, draw as many rectangles as you can that have the same perimeter as rectangle OABC.
c) Describe the patterns in the rectangles.
d) In part b, describe the rectangle with:
i) the maximum area
ii) the minimum area
e) Suppose you drew the rectangles in part b on plain paper. Would your answers to part d change? Explain.

6. **a)** For each perimeter below, determine the dimensions of the rectangle with the maximum area.
i) 8 cm **ii)** 34 cm **iii)** 54 cm
b) Calculate the area of each rectangle in part a.
How do you know each area is a maximum?

7. Steve has 58 m of fencing for a rectangular garden.
a) What are the dimensions of the largest garden Steve can enclose?
b) What is the area of this garden?

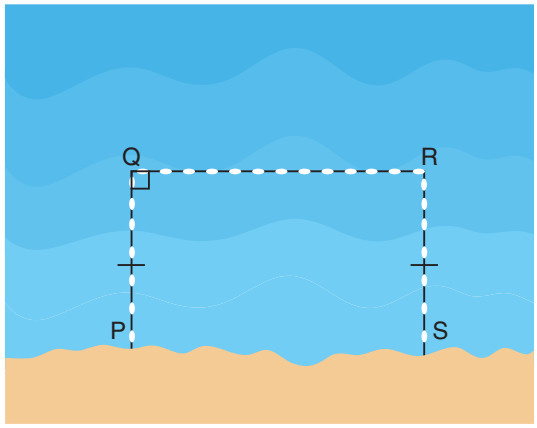
8. Refer to question 7.
Suppose the fencing comes in 1-m lengths that cannot be cut.
a) What are the dimensions of the largest garden Steve can enclose?
b) What is the area of this garden?

2.4 **9.** **a)** For each area below, determine the dimensions of a rectangle with the minimum perimeter.
i) 16 m^2 **ii)** 24 m^2 **iii)** 50 m^2
b) Calculate the perimeter of each rectangle in part a.
How do you know each perimeter is a minimum?

10. Determine the dimensions of a rectangle with area 196 cm^2 and the minimum perimeter. What is the minimum perimeter?

11. Rachel plans to build a rectangular patio with 56 square stones.
a) Sketch the different patios Rachel can create. Label each patio with its dimensions.
b) Suppose Rachel decides to put edging around the patio.
Which patio requires the minimum amount of edging?
Justify your answer.

2.5 **12.** A lifeguard has 400 m of rope to enclose a rectangular swimming area. One side of the rectangle is the beach, and does not need rope.



a) Suppose side QR measures 300 m. What is the length of RS?
b) Suppose side RS measures 85 m. What is the length of QR?
c) What are the dimensions of the rectangle with the maximum area? Justify your answer.

Practice Test

Multiple Choice: Choose the correct answer for questions 1 and 2.

1. Pina makes a rectangular garden surrounded by 32 m of edging.
Which of these figures has the maximum area?

A. A 2-m by 4-m rectangle
B. A 4-m by 12-m rectangle
C. A 6-m by 10-m rectangle
D. A square with side length 8 m

2. A rectangle has an area of 36 m^2.
Which dimensions will ensure that the rectangle has the minimum perimeter?

A. 6 m by 6 m
B. 9 m by 4 m
C. 12 m by 3 m
D. 18 m by 2 m

Show all your work for questions 3 to 7.

3. **Application** Lindsay was planning to build a rectangular patio with 24 square stones and to edge it on all four sides.

a) Sketch the different patios Lindsay can create.
Label each patio with its dimensions.

b) Which patio requires the minimum amount of edging?
Justify your answer.

4. **Knowledge and Understanding** Determine the maximum area of a rectangle with each given perimeter.

a) 28 cm
b) 86 cm

5. **Communication** When you see a rectangle, how do you know if:

a) it has the minimum perimeter for its area?

b) it has the maximum area for its perimeter?

Include diagrams in your explanation.

6. **Thinking** Eighteen sections of fence, each 1 m long, are used to make a rectangular storage area.

a) How many sections should be used for the length and the width to make the storage area a maximum?
What is this area?

b) Suppose workers found 2 more sections.
Where should these sections be put for the maximum storage area? Justify your answer.

7. Suppose the storage area in question 6 is bordered by a wall on one side and 18 sections around the remaining 3 sides. What are the dimensions of the largest area that can be formed?